Large Scale Aerodynamic Shape Optimization

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Multilevel Parameterizations and Fast Multigrid Methods for Aerodynamic Shape Optimization

DFG SPP 1253 - Optimization with Partial Differential Equations

June 4, 2009

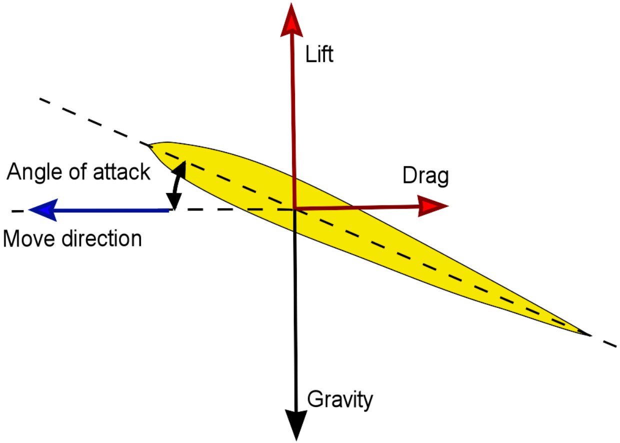
DLR

Universität Trier

Outline

1. Problem Introduction and Standard Parametric Paradigm
2. Shape Optimization Paradigm
   - Shape Derivatives
   - Shape Hessians for Stokes and Navier-Stokes Equations
   - Operator Symbol Approximations
3. Shape Optimization of Transonic and Supersonic Airfoils
   - Gradient Derivation
   - Preconditioning and One-Shot Optimization
4. 3D Flying Wing Optimization
5. Summary and Future Work

Introduction



Standard Parametric Paradigm: State of the Art

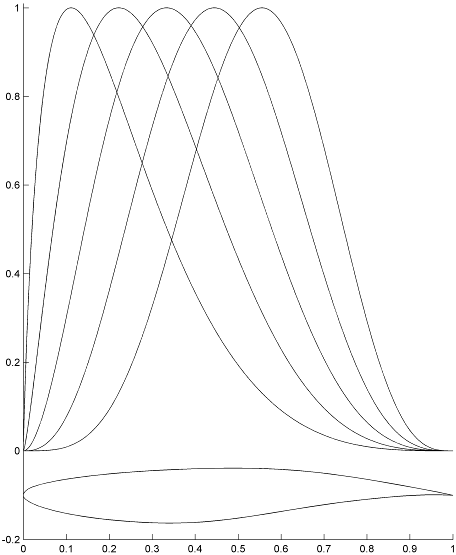
- Chose fixed geometry parametrization $q \in \mathbb{R}^{n_q}$
- Results in finite dimensional NLP:

$$\min_q F_\ell(u(q), q)$$

- Gradient given by formal Lagrangian for finite dimensional problem:

$$\frac{dF_\ell}{dq}(u(q), q) = \frac{\partial F_\ell}{\partial q} - \lambda^T \frac{\partial c}{\partial q}$$

$$\left[\frac{\partial c}{\partial u}\right]^T \lambda = \frac{\partial F_\ell}{\partial u}$$



Standard Parametric Paradigm: State of the Art

Pros:

- Easy to realize (given adjoint solver)
- Small $\frac{\text{optimization}}{\text{simulation}}$ for low dimensions (one-shot/SAND, ...)

Cons:

- Shape structure fixed
- Bad scaling with number of unknowns
- Mesh Sensitivity $\frac{\partial c}{\partial q}$ deteriorates with number of design variables

$$\frac{dF_\ell}{dq}(u(q), q) = \frac{\partial F_\ell}{\partial q} - \lambda^T \frac{\partial c}{\partial q}$$

Solution: Exploit Shape Optimization problem structure!
Wall-Clock-Time reduced by 99% (2D opt: 2.77h vs. 100s, 3D grad: 2.5d vs. 60s)!

The Hadamard Theorem

Under some regularity assumptions there exists a scalar distribution $G(\Gamma)$ with support on Γ such that

$$dJ(\Omega)[V] = \langle G(\Gamma), \langle V, n \rangle \rangle = \int_\Gamma \langle V, n \rangle \, g \, dS$$

- Shape Derivative is a scalar product with direction $\langle V, n \rangle$
- One evaluation of g per mesh node

Shape Derivative with Hadamard Theorem

$$J(\Omega) = \int_\Gamma h \, d\lambda^{n-1}$$

$$dJ(\Omega)[V] = \int_\Gamma \langle V(0), n \rangle \left[\frac{\partial h}{\partial n} + \kappa \, h \right] \, d\lambda^{n-1}$$

Observations

Pros:

- Gradient computation independent of design parameters! No mesh sensitivities
- No a priory geometry structure
- Mesh hierarchy defines shape hierarchy
- No mesh deformation, works with any PDE/flow solver

Cons:

- Iterate Ω_k can only be expressed in terms of Ω_{k-1}
  No design vector: $\Omega_k \neq \Omega_0(q_k)$ for $q_k \in \mathbb{R}^n$
  No NLP structure!
- Loss of regularity limits step length
- Hessian update formulas? One-shot?

Model Problems for Shape Hessians

Objective

$$\min_{(u,p,\Omega)} \dot{E}(u,\Omega) := \frac{1}{2} \int_\Omega \nu \sum_{j,k=1}^{3} \left(\frac{\partial u_k}{\partial x_j} \right)^2 dA$$

Constraints

a) Stokes Equation

$$\begin{aligned} -\nu \Delta u + \nabla p &= 0 \\ \text{div } u &= 0 \end{aligned}$$

b) Navier-Stokes Equation

$$\begin{aligned} -\nu \Delta u + \rho u \nabla u + \nabla p &= 0 \\ \text{div } u &= 0 \end{aligned}$$

+ volume constraint

First Order Shape Calculus

a) Stokes

$$d\dot{E}_S(u,\Omega)[V] = \int\limits_{\Gamma} \langle V, n\rangle \left[-\nu \sum_{k=1}^{3} \left(\frac{\partial u_k}{\partial n}\right)^2\right] \, dS$$

b) Navier-Stokes

$$d\dot{E}_{NS}(u,\Omega)[V] = \int\limits_{\Gamma} \langle V, n\rangle \left[-\nu \sum_{k=1}^{3} \left(\frac{\partial u_k}{\partial n}\right)^2 - \frac{\partial u_k}{\partial n}\frac{\partial \lambda_k}{\partial n}\right] \, dS$$

$$\begin{aligned} -\nu\Delta\lambda - \rho\lambda\nabla u - \rho\left(\nabla\lambda\right)^T u + \nabla\lambda_p &= -2\Delta u \quad \text{in } \Omega \\ \text{div } \lambda_p &= 0 \qquad\quad \text{in } \Omega \end{aligned}$$

Second Order Shape Calculus

$$d^2 \dot{E}_S(u,\Omega)[V,W] = I_1 + I_2$$

where

$$I_1 = \int_\Gamma \langle W, n\rangle \left[\operatorname{div} V \left(-\nu \sum_{i,j=1}^{3} \left(\frac{\partial u_i}{\partial x_j} \right)^2 \right) + V_\Gamma \nabla \left(-\nu \sum_{i,j=1}^{3} \left(\frac{\partial u_i}{\partial x_j} \right)^2 \right) \right] dS$$

$$I_2 = \int_\Gamma \langle V, n\rangle \left[2\nu \sum_{i=1}^{3} \frac{\partial u_i}{\partial n} S \left(\frac{\partial u_i}{\partial n} \langle W, n\rangle \right) \right] + \langle W, n\rangle \langle V, n\rangle \frac{\partial}{\partial n} \left(-\nu \sum_{i,j=1}^{3} \left(\frac{\partial u_i}{\partial x_j} \right)^2 \right) dS$$

- Divergence-free Poincaré-Steklov operator S
- I_1 hard to discretize
- I_2 has nullspace away from optimum

Symbol of an Operator

Suppose Fourier disturbance (oscillation) of design: $\tilde{q}(x) = e^{i\omega x}$

- First order differential operator: $H\tilde{q} = i\omega\tilde{q}$
- Second order differential operator: $H\tilde{q} = -\omega^2\tilde{q}$
- Dirichlet to Neumann Map / Poincaré-Steklov: $H\tilde{q} = |\omega|\tilde{q}$

Stokes (analytic) / Navier-Stokes (frequency analysis):

$$H\tilde{q} = (\beta \cdot |\omega| + \gamma)\tilde{q}$$

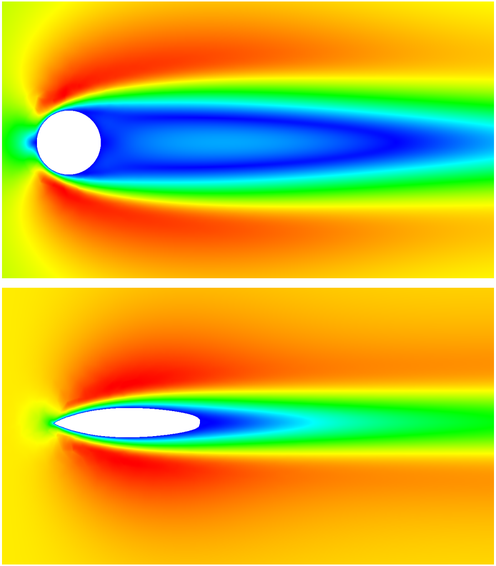
Approximation:

$$\tilde{H} = -\alpha\Delta_\Gamma + id$$

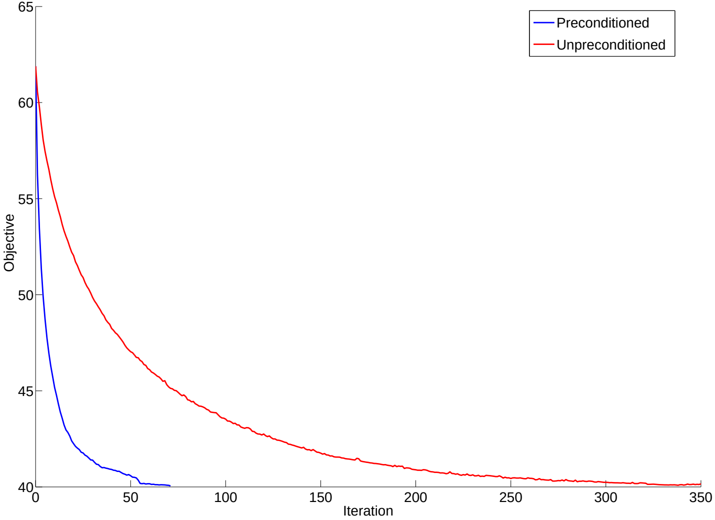
Symbol: $1 + \alpha\omega^2$

α chosen to match boundary discretization

Navier-Stokes: Initial and Optimal Domain



Performance: Navier-Stokes



- Optimum in iteration 71 vs 350: 80% less iterations

Euler Drag Reduction

Minimize Wave Drag

$$\min_{(u,\Omega)} J(u,\Omega) := \int_\Gamma \langle p_\ell, n \rangle \, dS = \int_\Gamma p \cdot n_\ell \, dS$$

subject to

$$\begin{aligned} 0 &= A_1(V)\frac{\partial U}{\partial x_1} + A_2(V)\frac{\partial U}{\partial x_2} + A_3(V)\frac{\partial U}{\partial x_3} \quad \text{in } \Omega \\ 0 &= \langle u, n \rangle \quad \text{on } \Gamma \\ u_\infty &= u \quad \text{on } \Gamma_{\text{inflow}} \end{aligned}$$

- Euler Flux Jacobian: $A_i(V)$
- Conserved variables: $U = (\rho, \rho u_1, \rho u_2, \rho u_3, \rho E)^T$
- Primitive variables: $V = (\rho, u_1, u_2, u_3, p)^T$
- Perfect gas: $p = (\gamma - 1)\rho(E - \frac{1}{2}(u_1^2 + u_2^2 + u_3^2))$

Euler Gradient

Shape Derivative for Euler Drag Reduction

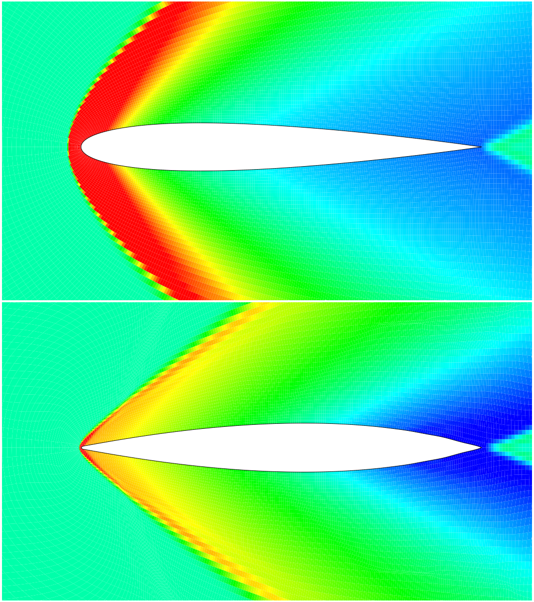
$$dF_\ell(\Omega)[V] = \int_\Gamma \langle V, n\rangle \left[\langle \nabla p_\ell\, n, n\rangle + \kappa \langle p_\ell, n\rangle - \lambda U_H \langle Du \cdot n, n\rangle\right] + (p_\ell - \lambda U_H u) dn[V]\, dS$$

$$= \int_\Gamma \langle V, n\rangle \left[\langle \nabla p_\ell\, n, n\rangle - \lambda U_H \langle Du \cdot n, n\rangle + \mathrm{div}_\Gamma\, (p_\ell - \lambda U_H u)\right]$$

- Hessian Symbol: 2D: $H\tilde{q} = -\omega^2 \tilde{q}$, 3D: $H\tilde{q} = -\frac{\omega_1^2}{\omega_2}\tilde{q}$
- MDO: Constraint on contour length and bending stiffness

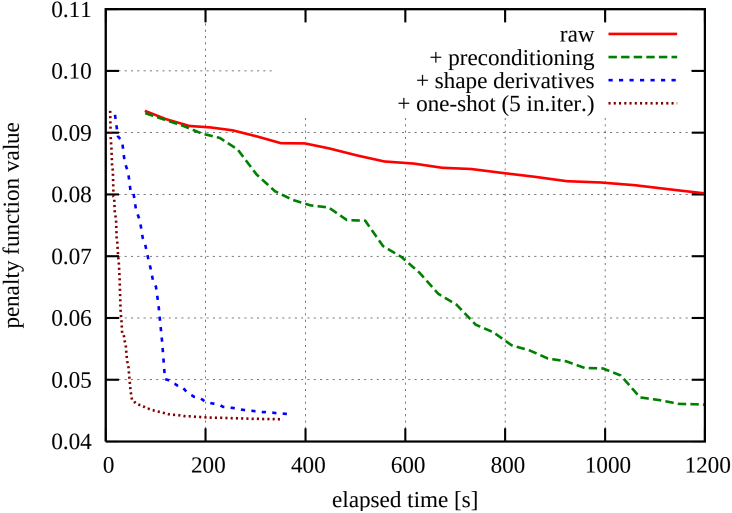
$$\int_\Gamma dS \leq L_0, \quad \int_\Gamma (y - y_c)^2\ dS \geq I_{x_0}$$

Optimized Shape: Supersonic



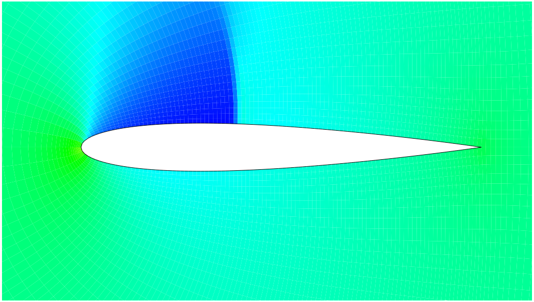
- DLR Flow Solver TAU
- Unstructured Finite Volume
- Mach 2.00
- Initial NACA0012: $C_D = 9.430 \cdot 10^{-2}$
- Optimal Haack Ogive: $C_D = 4.721 \cdot 10^{-2}$
- Reduction by 49.9%
- 400 design parameters

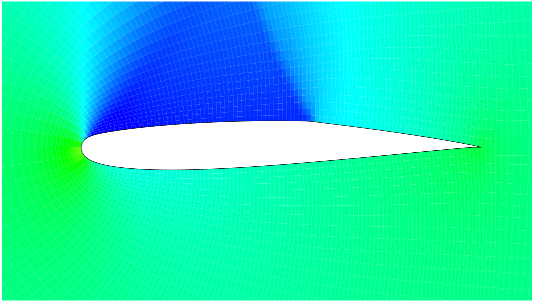
Optimization History: Wall-Clock-Time



Exploit shape optimization problem structure! Wall-clock time reduced by 99% (2.77h vs. 100s)

Optimized Shape: Transonic, Lifting





- Initial NACA0012
- Mach 0.73, AoA: 2°

$$C_D = 6.360 \cdot 10^{-3}$$
$$C_L = 4.020 \cdot 10^{-1}$$
$$V = 8.1341 \cdot 10^{-2}$$

- Constraints:

$$C_L = 8.171 \cdot 10^{-1}$$
$$V = 8.223 \cdot 10^{-2}$$

- Optimized, 60 Iterations:

$$C_D = 3.347 \cdot 10^{-3}$$
$$C_L = 8.174 \cdot 10^{-1}$$
$$V = 8.2185 \cdot 10^{-2}$$

- 200 Design Parameter

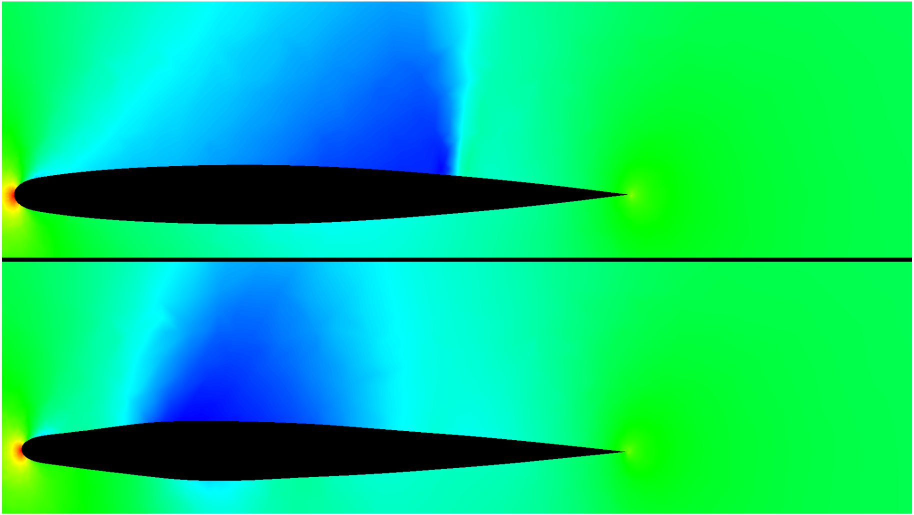
3D Flying Wing Optimization: Onera M6

	Shape	State	C_D	C_L	α	M_∞
M 1	18, 285	541, 980	$1.057 \cdot 10^{-2}$	$2.761 \cdot 10^{-1}$	3.01	0.83
M 2	36, 806	1, 486, 315				

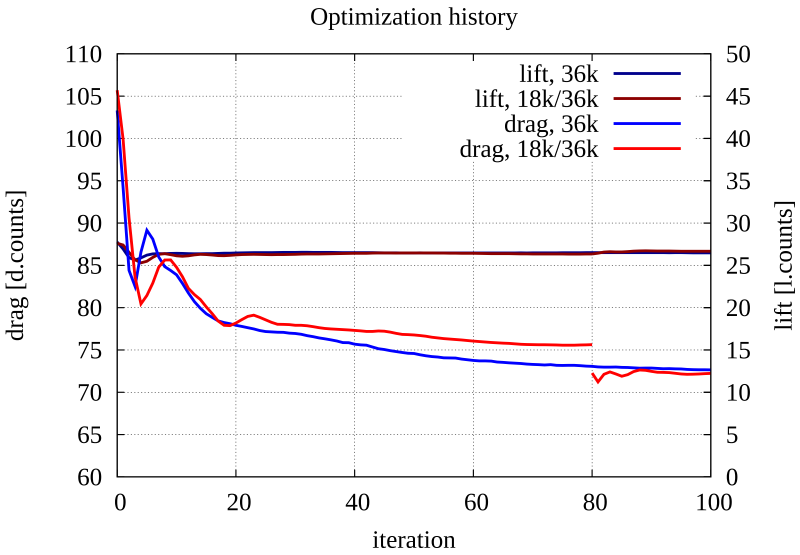
3D Flying Wing Optimization: Onera M6

	Shape	C_D	%	C_L	%	
M 1	18,285	$7.52 \cdot 10^{-3}$	-28.8%	$2.65 \cdot 10^{-1}$	-4.0%	$\approx$ 1 mins
M 2	36,806	$7.27 \cdot 10^{-3}$	-29.7%	$2.65 \cdot 10^{-1}$	-4.1%	$\approx$ 3 mins

3D Results



Multilevel Convergence History



Outlook and Future Work

Summary:

- Good Hessian approximation results in equation $\frac{\text{optimization}}{\text{simulation}} \approx 2.5$
- Knowledge of Hessian symbol has potential for mesh independent performance
- Structure exploitation CPU wall-clock time improvements:
  - Shape Hessian: 88%
  - Shape derivative: 75%

Outlook:

- 3D RANS-equation with turbulence modeling
- Fuselage
- GPU-solver