Aerodynamic Shape Optimization under Uncertainty

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Outline

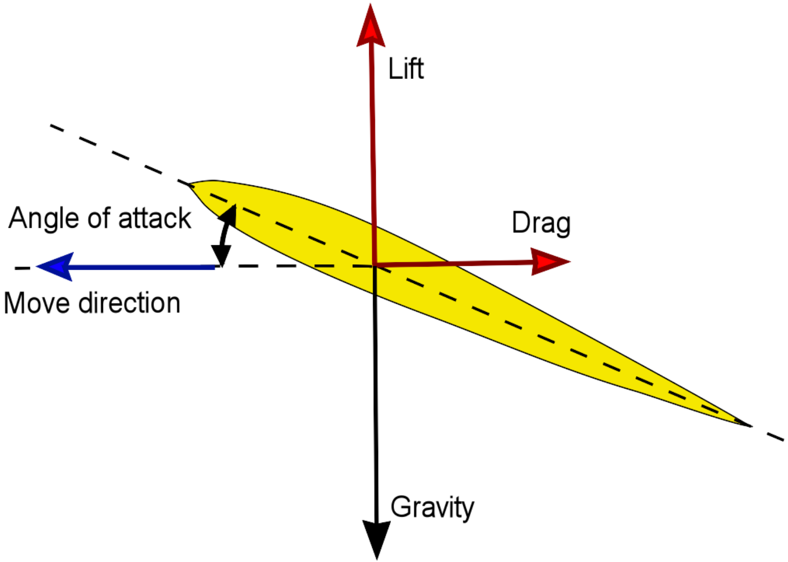
Motivation

- Aerodynamic performance of an airplane is very sensitive to the wing shape and flight conditions

  $\longrightarrow$ **Robust design**:
  optimality and robustness of performance against any uncertainties

- The resulting optimization tasks become much more complex than in the usual single setpoint case

  $\longrightarrow$ Development of efficient and fast algorithms


Lift
Angle of attack
Drag
Move direction
Gravity

Uncertainty sources

Uncertainties with respect to the flight conditions

- Mach number
- angle of attack
- density
- Reynolds number

Geometry uncertainties

Uncertainty modeling

scalar-valued uncertainties

- real-valued, continuous random variable $s : \Omega \rightarrow \mathbb{R}$ defined on a probability space (Ω, Y, P)
- characterized by a probability density function $\varphi : \mathbb{R} \rightarrow \mathbb{R}_+$

spatially distributed uncertainties

- random field $s : \Gamma \times \Omega \rightarrow \mathbb{R}$ defined on a probability space (Ω, Y, P)
- determined by its second-order statistics, i.e. by its mean and by its covariance function

Robust Design

single setpoint aerodynamic shape optimization problem

$$\min_{y,p} f(y,p)$$

$$\text{s.t. } c(y,p) = 0$$

$$h(y,p) \geq 0$$

one-shot approaches, e.g in [Gherman, Schulz 2007]

Robust Design

optimization problem influenced by stochastic perturbations $s : \Omega \to \mathbb{R}$, $\zeta \in \Omega$

$$\min_{y,p} f(y,p,\zeta)$$

$$\text{s.t. } c(y,p,\zeta) = 0$$

$$h(y,p,\zeta) \geq 0$$

- min-max formulation (not discussed here)
- semi-infinite formulation
- chance-constraint formulation

semi-infinite formulation (cf. Marti 2005)

optimizing the average objective function

$$\min_{y,p} \int_{\Omega} f(y,p,\zeta)\,d\mathcal{P}(\zeta)$$

$$\text{s.t. } c(y,p,\zeta) = 0\,,\ \forall \zeta \in \Omega$$

$$h(y,p,\zeta) \geq 0\,,\ \forall \zeta \in \Omega$$

semi-infinite formulation

discretization leads to

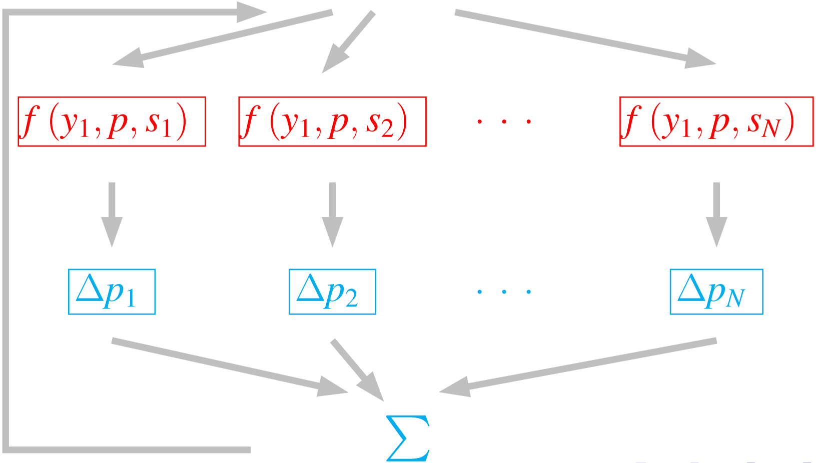
$$\min_{y_i,p} \sum_{i=1}^{N} f(y_i,p,s_i)\omega_i$$

$$\text{s.t. } c(y_i,p,s_i) = 0\,,\ \forall i \in \{1,\ldots,N\}$$

$$h(y_{min},p,s_{min}) \geq 0$$

Optimization strategy

cf.[Bock, Egartner, Kappis, Schulz 2002]



chance-constraint formulation (cf. Prekopa 1970, Henrion 2007)

inequality constraint hold with a certain probability $\mathcal{P}_0$

$$\min_{y,p} \int_\Omega f(y,p,\zeta)\,d\mathcal{P}(\zeta)$$

$$\text{s.t. } c(y,p,\zeta) = 0\,,\ \forall \zeta \in \Omega$$

$$\mathcal{P}(\{\zeta \,|\, h(y,p,\zeta) \geq 0\}) \geq \mathcal{P}_0$$

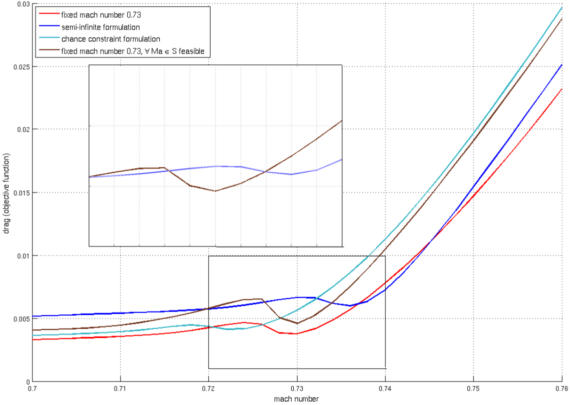
linearized chance-constraint formulation

Taylor-series approximation gives

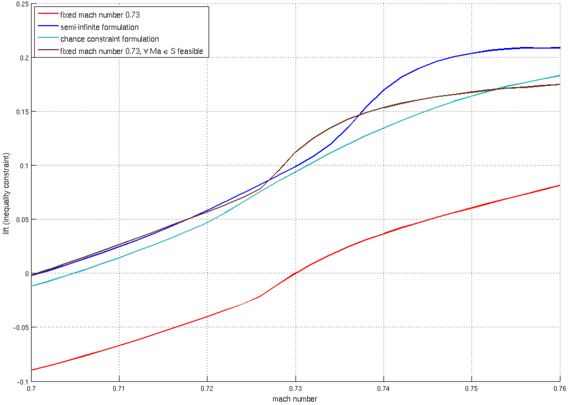
$$\min_p f(y(p,s^0),s^0) + \frac{1}{2}\frac{\partial^2 f(y(p,s^0),s^0)}{\partial \zeta^2} Var(s)$$

$$\text{s.t.} \quad \mathcal{P}(\{\zeta \,|\, h(s^0) + \frac{\partial h(s^0)}{\partial \zeta}(s(\zeta) - s^0) \geq 0\}) \geq \mathcal{P}_0$$

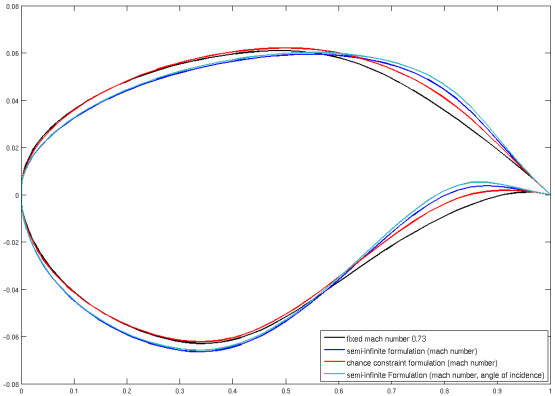
Comparison of drag



Comparison of lift



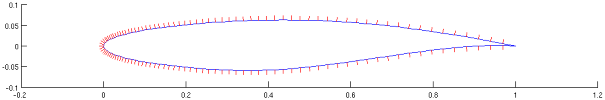
Optimized airfoil



Geometry uncertainties

Description of the geometry uncertainties

$$h(x,\omega) = x + s(x,\omega) \cdot \vec{n}(x) \qquad \forall x \in \Gamma, \omega \in \mathcal{Q}$$



Assumptions on the random field $s(x,\omega)$

- $\mathbb{E}(s(x,\zeta)) = s_0(x) = 0$
- $Cov(x,y) = b^2 \cdot \exp\left(-\frac{\|x-y\|^2}{l^2}\right) \qquad \forall x,y \in \Gamma$

Karhunen-Loève Expansion

truncated Karhunen-Loève Expansion

$$s_N(x,\omega) = s_0(x) + \sum_{j=1}^{N} \sqrt{\lambda_j} z_j(x)\, Y_j(\omega)$$

with λ_j eigenvalue, $z_j(x)$ eigenfunction of Cov

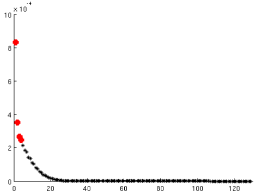
i.e. $\int_\Gamma Cov(x,y)\, z_j(y)\, dy = \lambda_j z_j(x)\,, \qquad x \in \Gamma$

error analysis

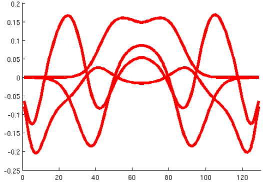
$$\lim_{N\to\infty} \left\{ \sup_\Gamma \int_Q (s - s_N)^2\, d\mathcal{P}(\omega) \right\} = \lim_{N\to\infty} \left\{ \sup_\Gamma \left(\sum_{j=N+1}^{\infty} \lambda_j z_j^2 \right) \right\}$$

Geometries

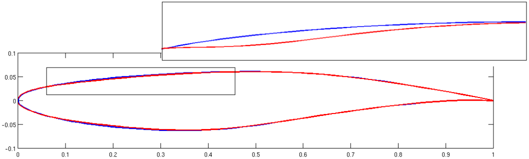
Eigenvalues



Eigenvectors



one perturbed geometry

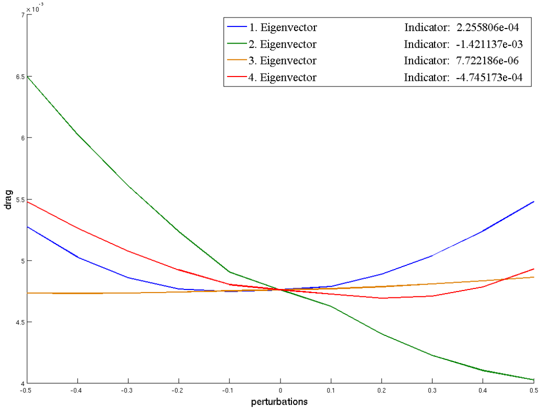


adaptive choice of the stochastic ansatz space

goal-oriented choice of the Karhunen-Loève basis
$\longrightarrow$ **gradient based indicator**

- Estimation of the approximation error using derivatives
- Error indicator for the individual eigenvectors
- automatic selection of the reduced basis

Importance of the individual basis vectors for the target functional



Sparse grid

Smolyak-Algorithm

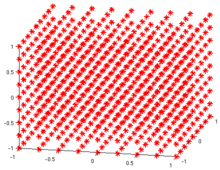
Idea: Combination of quadrature formulas of high order in only some dimensions and formulas of lower order in the other dimensions

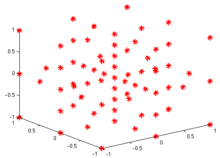
$$Q(k,d) = \sum_{|\mathbf{i}| \leq k} \Delta^{(i_1)} \otimes \cdots \otimes \Delta^{(i_d)}$$

$Q^{(1)}, Q^{(2)}, ...$ sequence of quadrature formulas

$\Delta^{(i)} = Q^{(i+1)} - Q^{(i)}$

$\mathbf{i} = (i_1, ..., i_d)\,, \quad |\mathbf{i}| = \sum_{\nu=1}^{d} i_\nu$

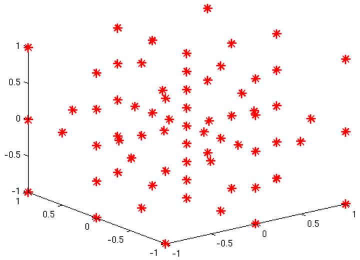




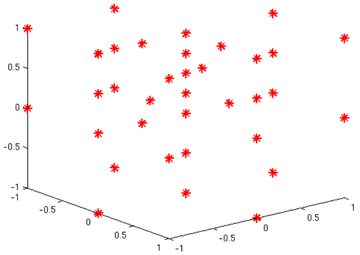
adaptive Sparse grid

cf.[Garcke, Griebel 2001]
automatic detection of important dimensions
→ **local refinement**

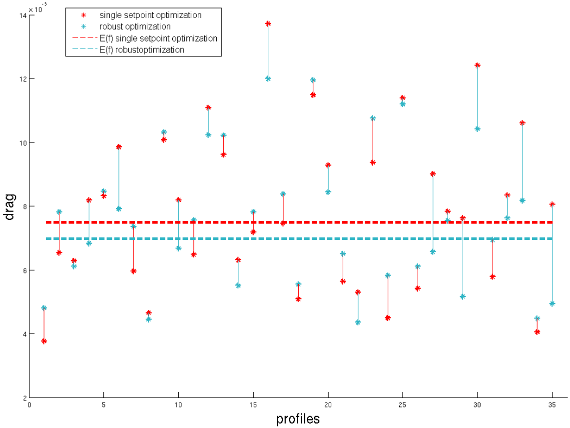
69 grid points



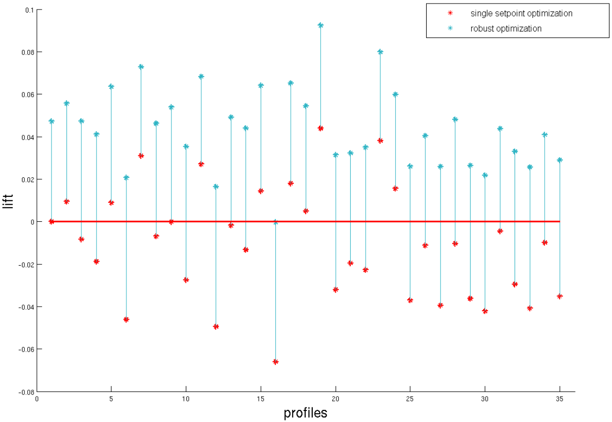
35 grid points



Drag performance of perturbed shapes

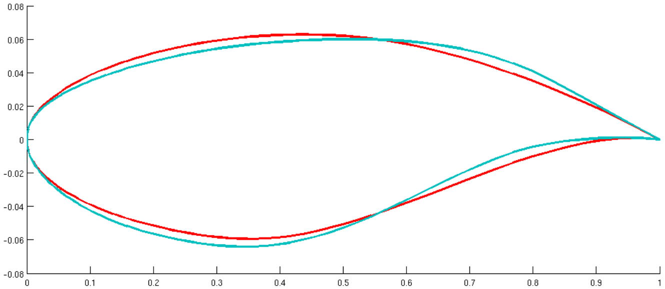


Lift performance of perturbed shapes



Optimized airfoil

Comparison of the robust shape and the shape resulting from the single setpoint optimization



Conclusions and further work

- several approaches to robust design under stochastic uncertainties are discussed
- the discretized semi-infinite approach seems most promising so far
- the very high dimensional optimization tasks are efficiently approached by the use of adaptive sparse grids and efficient parallelization of one-shot methods

- applying of the introduced methods to a 3D test case