On the Existence of Optimal Worst-Case Robust Controls

Frank Schmidt[1]

[1]Chemnitz University of Technology, Faculty of Mathematics
Research Group Numerical Mathematics (Partial Differential Equations)

CHEMNITZ UNIVERSITY
OF TECHNOLOGY

GAMM Workshop on PDE Constrained Optimization of Certain and Uncertain Processes, Trier

June 5, 2009

Outline

1. Motivation
2. Problems without Robust State Constraints
3. Problems with Robust State Constraints

Motivation I: Uncertain Coefficient

$$\min_{\substack{u\in L^2(\Gamma)\\ y\in H^1(\Omega)}} \quad \frac{1}{2}\|y-y_d\|^2_{L^2(\Omega)} + \frac{\gamma}{2}\|u\|^2_{L^2(\Gamma)}$$

$$\text{s.t.} \quad \begin{aligned} -\triangle y &= 0 && \text{in } \Omega \\ \partial_\nu y &= p\,(u-y) && \text{on } \Gamma \end{aligned}$$

$$\text{s.t.} \quad u_a \le u \le u_b \quad \text{a.e. on } \Gamma$$

Motivation I: Uncertain Coefficient

$$\min_{u \in L^2(\Gamma)} \quad \sup_{\substack{p \in P_{\mathrm{ad}} \\ y \in H^1(\Omega)}} \quad \frac{1}{2}\|y - y_d\|^2_{L^2(\Omega)} + \frac{\gamma}{2}\|u\|^2_{L^2(\Gamma)}$$

$$\text{s.t.} \quad -\triangle y = 0 \quad \text{in } \Omega$$

$$\partial_\nu y = p\,(u - y) \quad \text{on } \Gamma$$

$$\text{s.t.} \quad u_a \le u \le u_b \quad \text{a.e. on } \Gamma$$

Motivation I: Uncertain Coefficient

$$\min_{u \in L^2(\Gamma)} \quad \sup_{\substack{p \in P_{\text{ad}} \\ y \in H^1(\Omega)}} \quad \frac{1}{2}\|y - y_d\|^2_{L^2(\Omega)} + \frac{\gamma}{2}\|u\|^2_{L^2(\Gamma)}$$

$$\text{s.t.} \quad -\triangle y = 0 \quad \text{in } \Omega$$

$$\partial_\nu y = p\,(u - y) \quad \text{on } \Gamma$$

$$\text{s.t.} \quad u_a \leq u \leq u_b \quad \text{a.e. on } \Gamma$$

Reduced Formulation

$$\min_{u \in U_{\text{ad}}} \quad \sup_{p \in P_{\text{ad}}} \quad \frac{1}{2}\|S(u,p) - y_d\|^2_{L^2(\Omega)} + \frac{\gamma}{2}\|u\|^2_{L^2(\Gamma)}$$

where $S(u,p) \in H^1(\Omega)$ is the unique weak solution of the PDE

Motivation II: Perturbed Control

$$\min_{\substack{u\in L^2(\Omega)\\ y\in H^1(\Omega)}} \quad \frac{1}{2}\|y-y_d\|^2_{L^2(\Omega)} + \frac{\gamma}{2}\|u+p\|^2_{L^2(\Omega)}$$

$$\text{s.t.} \quad \begin{aligned} -\triangle y + y &= u + p && \text{in } \Omega \\ \partial_\nu y &= 0 && \text{on } \Gamma \end{aligned}$$

$$\text{s.t.} \quad u_a \le u \le u_b \quad \text{a.e. on } \Omega$$

Motivation II: Perturbed Control

$$\min_{u \in L^2(\Omega)} \quad \sup_{\substack{p \in P_{\text{ad}} \\ y \in H^1(\Omega)}} \quad \frac{1}{2}\|y - y_d\|^2_{L^2(\Omega)} + \frac{\gamma}{2}\|u + p\|^2_{L^2(\Omega)}$$

$$\text{s.t.} \quad \begin{aligned} -\triangle y + y &= u + p \quad &&\text{in } \Omega \\ \partial_\nu y &= 0 &&\text{on } \Gamma \end{aligned}$$

$$\text{s.t.} \quad u_a \leq u \leq u_b \quad \text{a.e. on } \Omega$$

Motivation II: Perturbed Control

$$\min_{u \in L^2(\Omega)} \quad \sup_{\substack{p \in P_{ad} \\ y \in H^1(\Omega)}} \quad \frac{1}{2}\|y - y_d\|^2_{L^2(\Omega)} + \frac{\gamma}{2}\|u + p\|^2_{L^2(\Omega)}$$

$$\text{s.t.} \quad -\triangle y + y = u + p \quad \text{in } \Omega$$

$$\partial_\nu y = 0 \quad \text{on } \Gamma$$

$$\text{s.t.} \quad u_a \leq u \leq u_b \quad \text{a.e. on } \Omega$$

Reduced Formulation

$$\min_{u \in U_{ad}} \quad \sup_{p \in P_{ad}} \quad \frac{1}{2}\|S(u,p) - y_d\|^2_{L^2(\Omega)} + \frac{\gamma}{2}\|u + p\|^2_{L^2(\Omega)}$$

where $S(u,p) \in H^1(\Omega)$ is the unique weak solution of the PDE

Motivation II: Perturbed Control

$$\min_{u \in L^2(\Omega)} \sup_{\substack{p \in P_{\text{ad}} \\ y \in H^1(\Omega)}} \frac{1}{2}\|y - y_d\|^2_{L^2(\Omega)} + \frac{\gamma}{2}\|u + p\|^2_{L^2(\Omega)}$$

$$\text{s.t.} \quad -\triangle y + y = u + p \quad \text{in } \Omega$$

$$\partial_\nu y = 0 \quad \text{on } \Gamma$$

$$\text{s.t.} \quad u_a \le u \le u_b \quad \text{a.e. on } \Omega$$

Reduced Formulation

$$\min_{u \in U_{\text{ad}}} \sup_{p \in P_{\text{ad}}} \frac{1}{2}\|S(u,p) - y_d\|^2_{L^2(\Omega)} + \frac{\gamma}{2}\|u + p\|^2_{L^2(\Omega)}$$

where $S(u,p) \in H^1(\Omega)$ is the unique weak solution of the PDE

Does there exists a worst-case robust optimal control?

Kinds of Robustness

Approximation problem $(A_0 + p\,A_1)\,u \approx b$

- **Nominal Problem**

$$\min_{u \in \mathbb{R}^n} \|A_0 u - b\|_2^2$$

Kinds of Robustness

Approximation problem $(A_0 + p\,A_1)\,u \approx b$

- **Nominal Problem**

$$\min_{u \in \mathbb{R}^n} \|A_0 u - b\|_2^2$$

- **Stochastic Robust Problem**

$$\min_{u \in \mathbb{R}^n} \mathbf{E}\|\,(A_0 + p\,A_1)\,u - b\|_2^2$$

where p is uniformly distributed on $[-1,\,1]$

Kinds of Robustness

Approximation problem $(A_0 + p\,A_1)\,u \approx b$

- **Nominal Problem**

$$\min_{u \in \mathbb{R}^n} \|A_0 u - b\|_2^2$$

- **Stochastic Robust Problem**

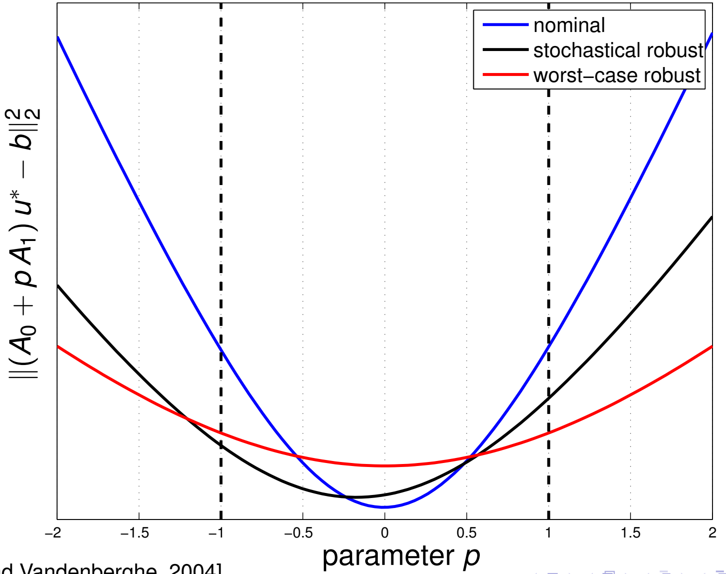
$$\min_{u \in \mathbb{R}^n} \mathbf{E}\|\,(A_0 + p\,A_1)\,u - b\|_2^2$$

where p is uniformly distributed on $[-1,\,1]$

- **Worst-case Robust Problem**

$$\min_{u \in \mathbb{R}^n} \max_{p \in [-1,1]} \|\,(A_0 + p\,A_1)\,u - b\|_2^2$$

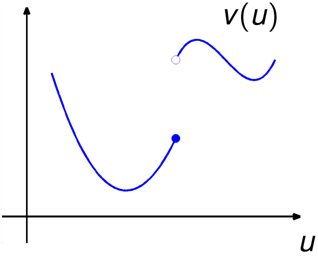
Comparison of Nominal, Stochastic and Worst-case



[Boyd and Vandenberghe, 2004]

Existence (finite- and infinite-dimensional)

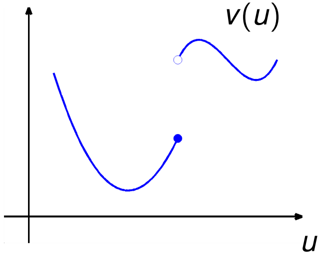
$$\min_{u \in U_{\mathrm{ad}}} v(u) \qquad (*)$$



Theorem

Let $\dim U_{\mathrm{ad}}$ *be finite.* $(*)$ *has a global minimizer, if*

$$\min_{u \in U_{ad}} v(u) \qquad (*)$$



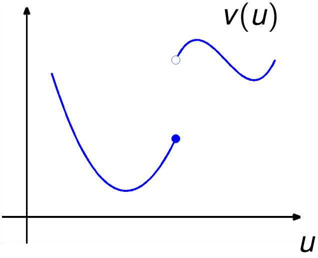
Theorem

Let $\dim U_{ad}$ be finite. $(*)$ has a global minimizer, if

- *U_{ad} is bounded*

Existence (finite- and infinite-dimensional)

$$\min_{u \in U_{\text{ad}}} v(u) \qquad (*)$$



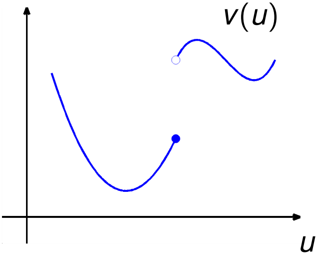
Theorem

Let $\dim U_{\text{ad}}$ *be finite.* $(*)$ *has a global minimizer, if*

- U_{ad} *is bounded*
- $v(u)$ *bounded from below*

Existence (finite- and infinite-dimensional)

$$\min_{u \in U_{\text{ad}}} v(u) \qquad (*)$$



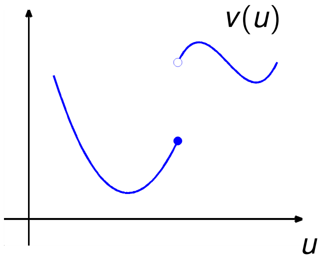
Theorem

Let $\dim U_{\text{ad}}$ *be finite.* $(*)$ *has a global minimizer, if*

- U_{ad} *is bounded*
- $v(u)$ *bounded from below*
- U_{ad} *is closed*

Existence (finite- and infinite-dimensional)

$$\min_{u \in U_{ad}} v(u) \qquad (*)$$



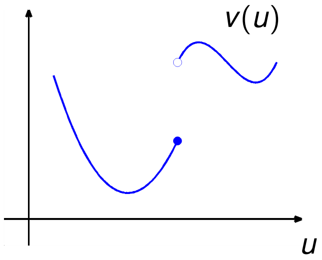
Theorem

Let $\dim U_{ad}$ be finite. $(*)$ has a global minimizer, if

- *U_{ad} is bounded*
- *$v(u)$ bounded from below*
- *U_{ad} is closed*
- *$v(u)$ is lower semicontinuous*

Existence (finite- and infinite-dimensional)

$$\min_{u \in U_{ad}} v(u) \qquad (*)$$



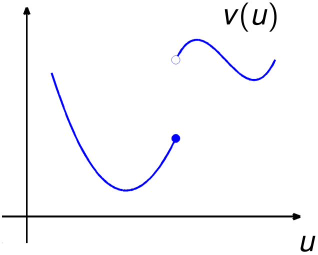
Theorem

Let $\dim U_{ad}$ be infinite. $(*)$ has a global minimizer, if

- U_{ad} *is bounded*
- $v(u)$ *bounded from below*
- U_{ad} *is closed*
- $v(u)$ *is lower semicontinuous*

Existence (finite- and infinite-dimensional)

$$\min_{u \in U_{\text{ad}}} v(u) \qquad (*)$$



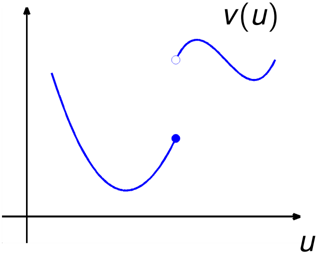
Theorem

Let $\dim U_{\text{ad}}$ *be infinite. $(*)$ has a global minimizer, if*

- U_{ad} *is bounded*
- $v(u)$ *bounded from below*
- U_{ad} *is weakly closed*
- $v(u)$ *is lower semicontinuous*

Existence (finite- and infinite-dimensional)

$$\min_{u \in U_{ad}} v(u) \qquad (*)$$



Theorem

Let $\dim U_{ad}$ *be infinite. $(*)$ has a global minimizer, if*

- U_{ad} *is bounded*
- $v(u)$ *bounded from below*
- U_{ad} *is weakly closed*
- $v(u)$ *is weakly lower semicontinuous*

Weak Lower Semicontinuity of OVF

Lemma

The Optimal Value Function

$$v(u) := \sup_{p \in P_{\text{ad}}} f(u,p)$$

is weakly lower semicontinuous if $u \mapsto f(u,p)$ *is weakly lower semicontinuous for all* $p \in P_{\text{ad}}$.

Existence of Worst-Case Robust Control

$$\min_{u\in U_{\text{ad}}} \quad \sup_{p\in P_{\text{ad}}} \quad \frac{1}{2}\|S(u,p) - y_d\|^2_{L^2(\Omega)} + \frac{\gamma}{2}\|u\|^2_{L^2(\Omega)}$$

where $S(u,p) \in H^1(\Omega)$ is the unique weak solution of

$$\begin{aligned} -\triangle y &= 0 && \text{in } \Omega \\ \partial_\nu y &= p\,(u - y) && \text{on } \Gamma \end{aligned}$$

Existence of Worst-Case Robust Control

$$\min_{u \in U_{\text{ad}}} \quad \sup_{p \in P_{\text{ad}}} \quad \underbrace{\frac{1}{2}\|S(u,p) - y_d\|^2_{L^2(\Omega)} + \frac{\gamma}{2}\|u\|^2_{L^2(\Omega)}}_{=f(u,p)}$$

where $S(u,p) \in H^1(\Omega)$ is the unique weak solution of

$$\begin{aligned} -\triangle y &= 0 && \text{in } \Omega \\ \partial_\nu y &= p\,(u - y) && \text{on } \Gamma \end{aligned}$$

Existence of Worst-Case Robust Control

$$\min_{u \in U_{\text{ad}}} \quad \sup_{p \in P_{\text{ad}}} \quad \underbrace{\frac{1}{2}\|S(u,p) - y_d\|^2_{L^2(\Omega)} + \frac{\gamma}{2}\|u\|^2_{L^2(\Omega)}}_{=f(u,p)}$$

where $S(u,p) \in H^1(\Omega)$ is the unique weak solution of

$$\begin{aligned} -\triangle y &= 0 && \text{in } \Omega \\ \partial_\nu y &= p\,(u - y) && \text{on } \Gamma \end{aligned}$$

Theorem

There exists a worst-case robust optimal control.

Proof.

□

Existence of Worst-Case Robust Control

$$\min_{u \in U_{\mathrm{ad}}} \quad \sup_{p \in P_{\mathrm{ad}}} \quad \underbrace{\frac{1}{2}\|S(u,p) - y_d\|^2_{L^2(\Omega)} + \frac{\gamma}{2}\|u\|^2_{L^2(\Omega)}}_{=f(u,p)}$$

where $S(u,p) \in H^1(\Omega)$ is the unique weak solution of

$$\begin{aligned} -\triangle y &= 0 && \text{in } \Omega \\ \partial_\nu y &= p\,(u - y) && \text{on } \Gamma \end{aligned}$$

Theorem

There exists a worst-case robust optimal control.

Proof.

- U_{ad} is closed, bounded and convex

□

Existence of Worst-Case Robust Control

$$\min_{u\in U_{\text{ad}}} \quad \sup_{p\in P_{\text{ad}}} \quad \underbrace{\frac{1}{2}\|S(u,p)-y_d\|^2_{L^2(\Omega)} + \frac{\gamma}{2}\|u\|^2_{L^2(\Omega)}}_{=f(u,p)}$$

where $S(u,p) \in H^1(\Omega)$ is the unique weak solution of

$$\begin{aligned} -\triangle y &= 0 && \text{in } \Omega \\ \partial_\nu y &= p\,(u-y) && \text{on } \Gamma \end{aligned}$$

Theorem

There exists a worst-case robust optimal control.

Proof.

- U_{ad} is closed, bounded and convex $\Longrightarrow$ U_{ad} is weakly closed

□

Existence of Worst-Case Robust Control

$$\min_{u\in U_{\text{ad}}} \quad \sup_{p\in P_{\text{ad}}} \quad \underbrace{\frac{1}{2}\|S(u,p)-y_d\|^2_{L^2(\Omega)} + \frac{\gamma}{2}\|u\|^2_{L^2(\Omega)}}_{=f(u,p)}$$

where $S(u,p) \in H^1(\Omega)$ is the unique weak solution of

$$\begin{aligned} -\triangle y &= 0 && \text{in } \Omega \\ \partial_\nu y &= p\,(u-y) && \text{on } \Gamma \end{aligned}$$

Theorem

There exists a worst-case robust optimal control.

Proof.

- U_{ad} is closed, bounded and convex $\Longrightarrow$ U_{ad} is weakly closed
- $S(u,p)$ is continuous w.r.t. u from $L^2(\Gamma)$ to $H^1(\Omega)$

□

Existence of Worst-Case Robust Control

$$\min_{u \in U_{\text{ad}}} \quad \sup_{p \in P_{\text{ad}}} \quad \underbrace{\frac{1}{2}\|S(u,p) - y_d\|^2_{L^2(\Omega)} + \frac{\gamma}{2}\|u\|^2_{L^2(\Omega)}}_{=f(u,p)}$$

where $S(u,p) \in H^1(\Omega)$ is the unique weak solution of

$$\begin{aligned} -\triangle y &= 0 && \text{in } \Omega \\ \partial_\nu y &= p\,(u - y) && \text{on } \Gamma \end{aligned}$$

Theorem

There exists a worst-case robust optimal control.

Proof.

- U_{ad} is closed, bounded and convex $\Longrightarrow$ U_{ad} is weakly closed
- $S(u,p)$ is continuous w.r.t. u from $L^2(\Gamma)$ to $H^1(\Omega)$ $\Longrightarrow$ $v(u) = \sup_{p \in P_{\text{ad}}} f(u,p)$ is weakly lower semicontinuous □

Outline

Integral State Constraints

$$\min_{u \in U_{\text{ad}}} \quad \sup_{p \in P_{\text{ad}}} \quad \frac{1}{2} \| S(u,p) - y_d \|^2_{L^2(\Omega)} + \frac{\gamma}{2} \| u \|^2_{L^2(\Gamma)}$$

Integral State Constraints

$$\min_{u \in U_{\text{ad}}} \sup_{p \in P_{\text{ad}}} \frac{1}{2}\|S(u,p) - y_d\|^2_{L^2(\Omega)} + \frac{\gamma}{2}\|u\|^2_{L^2(\Gamma)}$$

$$\text{s.t.} \quad g(\, S(u,p)\,) \leq 0 \quad \forall p \in P_{\text{ad}} \qquad g : L^2(\Omega) \to \mathbb{R}$$

Integral State Constraints

$$\min_{u \in U_{\mathrm{ad}}} \quad \sup_{p \in P_{\mathrm{ad}}} \quad \frac{1}{2}\|S(u,p) - y_d\|^2_{L^2(\Omega)} + \frac{\gamma}{2}\|u\|^2_{L^2(\Gamma)}$$

$$\text{s.t.} \quad g(\, S(u,p)\,) \le 0 \quad \forall p \in P_{\mathrm{ad}} \qquad g : L^2(\Omega) \to \mathbb{R}$$

Infinite-Dimensional Semi-Infinite Program

Integral State Constraints

$$\min_{u \in U_{\text{ad}}} \quad \sup_{p \in P_{\text{ad}}} \quad \frac{1}{2}\|S(u,p) - y_d\|^2_{L^2(\Omega)} + \frac{\gamma}{2}\|u\|^2_{L^2(\Gamma)}$$

$$\text{s.t.} \quad g(\, S(u,p)\,) \leq 0 \quad \forall p \in P_{\text{ad}} \qquad\qquad g : L^2(\Omega) \to \mathbb{R}$$

Infinite-Dimensional Semi-Infinite Program

Feasible Set

$$M = \{u \in U_{\text{ad}} : g(\, S(u,p)\,) \leq 0 \quad \forall p \in P_{\text{ad}}\}$$

Integral State Constraints

$$\min_{u \in U_{\text{ad}}} \quad \sup_{p \in P_{\text{ad}}} \quad \frac{1}{2}\|S(u,p) - y_d\|^2_{L^2(\Omega)} + \frac{\gamma}{2}\|u\|^2_{L^2(\Gamma)}$$

$$\text{s.t.} \quad g(\, S(u,p)\,) \leq 0 \quad \forall p \in P_{\text{ad}} \qquad\qquad g : L^2(\Omega) \to \mathbb{R}$$

Infinite-Dimensional Semi-Infinite Program

Feasible Set

$$\begin{aligned} M &= \{u \in U_{\text{ad}} : g(\, S(u,p)\,) \leq 0 \quad \forall p \in P_{\text{ad}}\} \\ &= U_{\text{ad}} \cap \bigcap_{p \in P_{\text{ad}}} \{u : g(\, S(u,p)\,) \leq 0\} \end{aligned}$$

Integral State Constraints

$$\min_{u \in U_{\text{ad}}} \quad \sup_{p \in P_{\text{ad}}} \quad \frac{1}{2}\|S(u,p) - y_d\|^2_{L^2(\Omega)} + \frac{\gamma}{2}\|u\|^2_{L^2(\Gamma)}$$

$$\text{s.t.} \quad g(\,S(u,p)\,) \le 0 \quad \forall p \in P_{\text{ad}} \qquad\qquad g : L^2(\Omega) \to \mathbb{R}$$

Infinite-Dimensional Semi-Infinite Program

Feasible Set

$$\begin{aligned} M &= \{u \in U_{\text{ad}} : g(\,S(u,p)\,) \le 0 \quad \forall p \in P_{\text{ad}}\} \\ &= U_{\text{ad}} \cap \bigcap_{p \in P_{\text{ad}}} \{u : g(\,S(u,p)\,) \le 0\} \end{aligned}$$

Lemma

The intersection of weakly closed sets is weakly closed.

Lemma

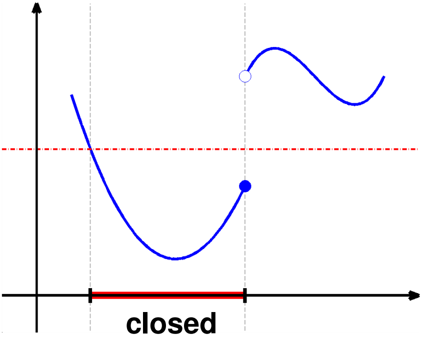
If

$$g : L^2(\Omega) \to \mathbb{R}$$

is lower semicontinuous then the set

$$\{u : g(\,S(u,p)\,) \leq 0\}$$

is weakly closed for a fixed p.



Integral State Constraints - Level Sets

Lemma

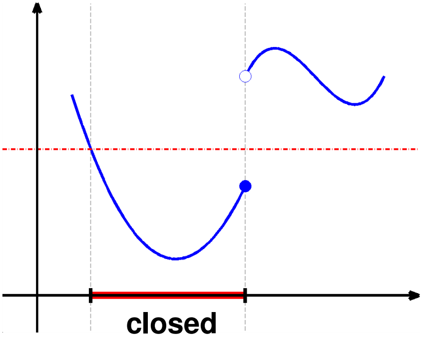
If

$$g : L^2(\Omega) \to \mathbb{R}$$

is lower semicontinuous then the set

$$\{u : g(\, S(u, p)\,) \leq 0\}$$

is weakly closed for a fixed p.



Theorem

There exists a worst-case robust optimal control $u^* \in U_{\text{ad}}$.

General State Constraints

$$\min_{u \in U_{\text{ad}}} \quad \sup_{p \in P_{\text{ad}}} \quad \frac{1}{2}\|S(u,p) - y_d\|^2_{L^2(\Omega)} + \frac{\gamma}{2}\|u\|^2_{L^2(\Gamma)}$$

$$\text{s.t.} \quad g(\, S(u,p)\,) \leq 0 \quad \forall p \in P_{\text{ad}}$$

General State Constraints

$$\min_{u \in U_{\text{ad}}} \quad \sup_{p \in P_{\text{ad}}} \quad \frac{1}{2}\|S(u,p) - y_d\|^2_{L^2(\Omega)} + \frac{\gamma}{2}\|u\|^2_{L^2(\Gamma)}$$

$$\text{s.t.} \quad G(\, S(u,p)\,) \preceq_K 0 \quad \forall p \in P_{\text{ad}}$$

General State Constraints

$$\min_{u \in U_{\text{ad}}} \quad \sup_{p \in P_{\text{ad}}} \quad \frac{1}{2}\|S(u,p) - y_d\|^2_{L^2(\Omega)} + \frac{\gamma}{2}\|u\|^2_{L^2(\Gamma)}$$

$$\text{s.t.} \quad G(\,S(u,p)\,) \preceq_K 0 \quad \forall p \in P_{\text{ad}}$$

Infinite-Dimensional Semi-Infinite Program

General State Constraints

$$\min_{u \in U_{\text{ad}}} \quad \sup_{p \in P_{\text{ad}}} \quad \frac{1}{2}\|S(u,p) - y_d\|^2_{L^2(\Omega)} + \frac{\gamma}{2}\|u\|^2_{L^2(\Gamma)}$$

$$\text{s.t.} \quad G(\,S(u,p)\,) \preceq_K 0 \quad \forall p \in P_{\text{ad}}$$

Infinite-Dimensional Semi-Infinite Program

Example (Pointwise State Constraints)

$$G : C(\overline{\Omega}) \to C(\overline{\Omega}),\ y \mapsto y - \psi \quad \text{and} \quad K = C^+(\overline{\Omega})$$

General State Constraints

$$\min_{u\in U_{\text{ad}}} \quad \sup_{p\in P_{\text{ad}}} \quad \frac{1}{2}\|S(u,p)-y_d\|^2_{L^2(\Omega)} + \frac{\gamma}{2}\|u\|^2_{L^2(\Gamma)}$$

$$\text{s.t.} \quad G(\,S(u,p)\,) \preceq_K 0 \quad \forall p \in P_{\text{ad}}$$

Infinite-Dimensional Semi-Infinite Program

Example (Pointwise State Constraints)

$$G : C(\overline{\Omega}) \to C(\overline{\Omega}),\; y \mapsto y - \psi \quad \text{and} \quad K = C^+(\overline{\Omega})$$

Feasible Set

$$M = U_{\text{ad}} \cap \bigcap_{p\in P_{\text{ad}}} \{u : G(\,S(u,p)\,) \preceq_K 0\}$$

General State Constraints

$$\min_{u \in U_{\text{ad}}} \quad \sup_{p \in P_{\text{ad}}} \quad \frac{1}{2}\|S(u,p) - y_d\|^2_{L^2(\Omega)} + \frac{\gamma}{2}\|u\|^2_{L^2(\Gamma)}$$

$$\text{s.t.} \quad G(\, S(u,p)\,) \preceq_K 0 \quad \forall p \in P_{\text{ad}}$$

Infinite-Dimensional Semi-Infinite Program

Example (Pointwise State Constraints)

$$G : C(\overline{\Omega}) \to C(\overline{\Omega}),\ y \mapsto y - \psi \quad \text{and} \quad K = C^+(\overline{\Omega})$$

Feasible Set

$$M = U_{\text{ad}} \cap \bigcap_{p \in P_{\text{ad}}} \{u : G(\, S(u,p)\,) \preceq_K 0\}$$

Theorem

There exists a worst-case robust optimal control $u^* \in U_{\text{ad}}$, if G is continuous

Summary and Outlook

1. Showed weak lower semicontinuity of optimal value function

Summary and Outlook

1. Showed weak lower semicontinuity of optimal value function

2. Proved existence of optimal worst-case robust controls for problems with and without state constraints

Summary and Outlook

1. Showed weak lower semicontinuity of optimal value function
2. Proved existence of optimal worst-case robust controls for problems with and without state constraints
3. Necessary optimality conditions $\rightsquigarrow$ MPCC

End

Thank You!