An efficient adjoint computation for flow control problems

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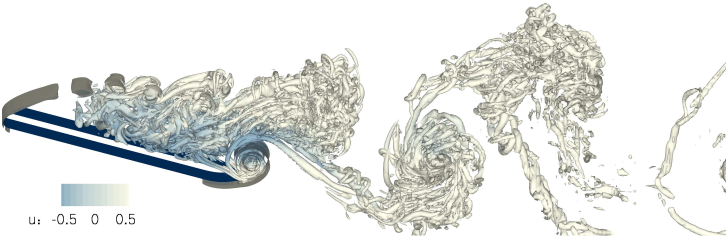
Outline

Motivation



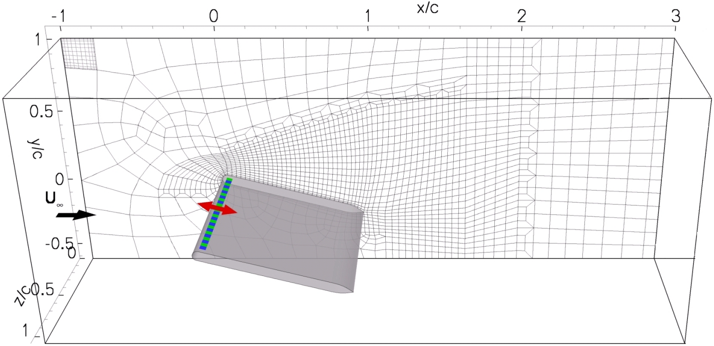
[T. Albrecht, J. Stiller, Proc. 3rd iTi Conf. Turbulence, 2008]

coupling advanced methods of physical numerical models and methods of mathematical control

➤ improvement of flow qualities

aim: control transient/turbulent flows with high Reynolds numbers

- complex physical background ➤ fine mesh



[T. Albrecht, J. Stiller, Proc. 3rd iTi Conf. Turbulence, 2008]

➤ appropriate small time steps

➤ for long-time simulation: high number of steps

- turbulence require a 3-D simulation

Semtex

- numerical simulation of fluid dynamics
- solve time dependent Navier-Stokes equations

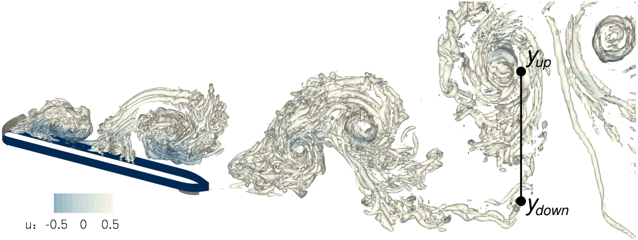
$$\frac{\delta u}{\delta t} + \nabla p = \nu \Delta u - (u \cdot \nabla) u + q$$

- spectral element method
  - high order finite element technique
  - combination of geometric flexibility of finite elements and high accuracy of spectral methods
- using:
  - parametrically mapped quadrilateral elements
  - Gauss-Lobatto-Legendre nodal shape function basis
  - continuous Galerkin projection

Modifications in Semtex

definition of a functional to minimize aerodynamic resistance

➔ minimize the momentum thickness using force q



$$J(x,q) = \sum_{n=1}^{N} \int_{t_{n-1}}^{t_n} \int_{y_{down}}^{y_{up}} \frac{u}{u_0}(1 - \frac{u}{u_0}) dy\, dt + \mu \|q\|^2 \rightarrow \text{min!}$$

$x = (u, v)$ $\quad u_0$ inflow velocity in horizontal direction

Flow control

optimization by using body forces

➤ high dimensional optimal control problem

➤ calculate gradients by using adjoint informations to reduce the effort

Adjoint calculation

different techniques: algorithmic differentiation
discretization of continuous adjoint
hand coded
...

Gradient Calculation (Time Integration)

1. Forward integration (flow simulation)

   $x_{i+1} = F_i(x_i, q_i), \; i = 1, \ldots, l$

   with $x_i \in \mathbb{R}^n$ state, q_i control at time t_i

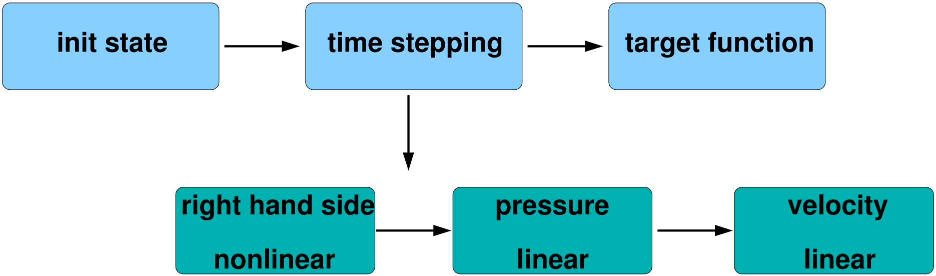
   number of time steps l known

2. Evaluation of **target function** $J(x, q)$

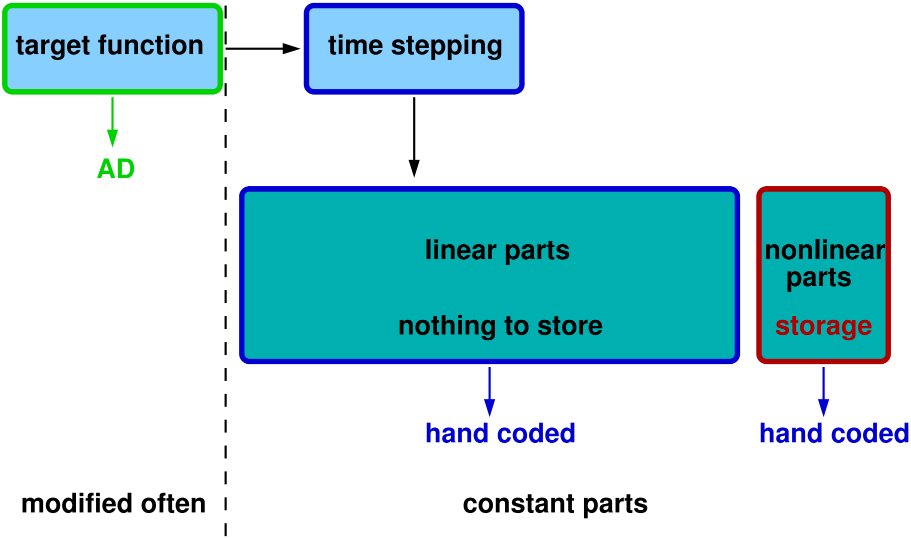
3. **Reverse integration**:

   $\bar{x}_{i-1} = \bar{F}_i(\bar{x}_i, \mathbf{x_{i-1}}, q_{i-1}, q_i), \; i = l, \ldots, 1$

4. Gradient calculation


init state
time stepping
target function
right hand side
nonlinear
pressure
linear
velocity
linear

Adjoint?


target function
time stepping
AD
linear parts
nothing to store
nonlinear parts
storage
hand coded
hand coded
modified often
constant parts

Algorithmic Differentiation (AD)

Differentiation of “computer programs” within machine precision

Basic forms: [Griewank/Walther 2008]

forward mode:	$\mathrm{OPS}(F'(x)\dot{x})$	$\leq$	$c_1\,\mathrm{OPS}(F), c_1 \in [2, 5/2]$
reverse mode:	$\mathrm{OPS}(\bar{y}^\top F'(x))$	$\leq$	$c_2\,\mathrm{OPS}(F), c_2 \in [3, 4]$
	$\mathrm{MEM}(\bar{y}^\top F'(x))$	$\sim$	$\mathrm{OPS}(F)$
combination:	$\mathrm{OPS}(\bar{y}^\top F''(x)\dot{x})$	$\leq$	$c_3\,\mathrm{OPS}(F), c_3 \in [7, 10]$

Tasks:

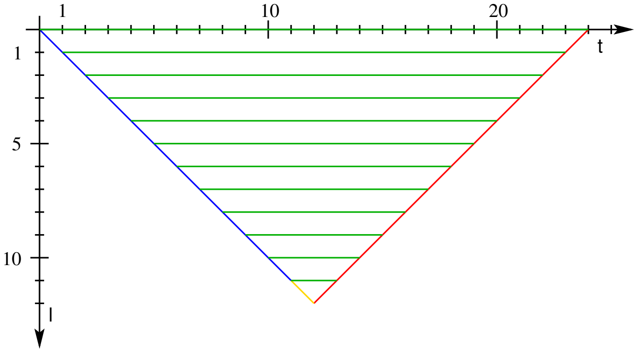
Derivatives (of any order)

Important topics:

Advanced AD?
(sparsity patterns, structure exploitation, time-stepping, . . .)

Store-Everything Approach

Example: 12 time steps



→ MEM = $O(l)$ $t = 2l$ **here:** #eval(F_i) = 12 t = 24

Binomial Checkpointing

Example: 12 time steps, 3 checkpoints and reusage of all checkpoints!

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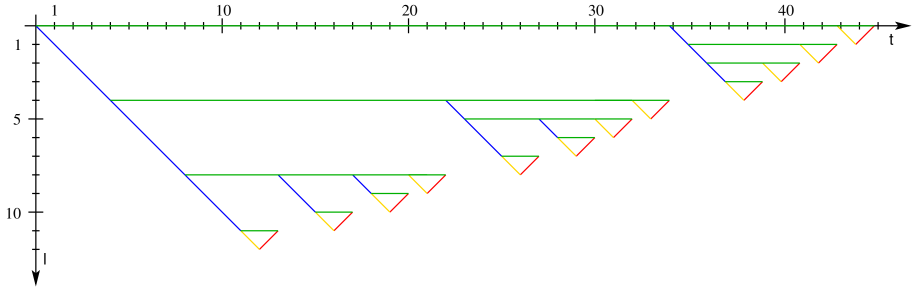


➔ MEM = c $t = ?$ **here:** c=3 #eval(F_i) = 21 t=45

Theorem (Complexity Result (Binomial Checkpointing))

Given

$$\begin{aligned} l &= \text{total number of time steps to be reversed} \\ c &= \text{number of checkpoints that can be used} \\ r &= \text{unique integer satisfying} \end{aligned}$$

$$\beta(c, r-1) \;<\; l \;\leq\; \beta(c, r) \;\equiv\; \binom{c+r}{c}$$

➔ *Minimal number of forward steps required is given by*

$$t(c, l) \;=\; rl - \beta(c+1, r-1) + 1$$

Proof: Based on induction (Griewank, Walther 2000)

Numerical results

simulation of flows with $Re = 10^4$:

- spectral element mesh with 1757 elements
- polynomial order $N = 12$

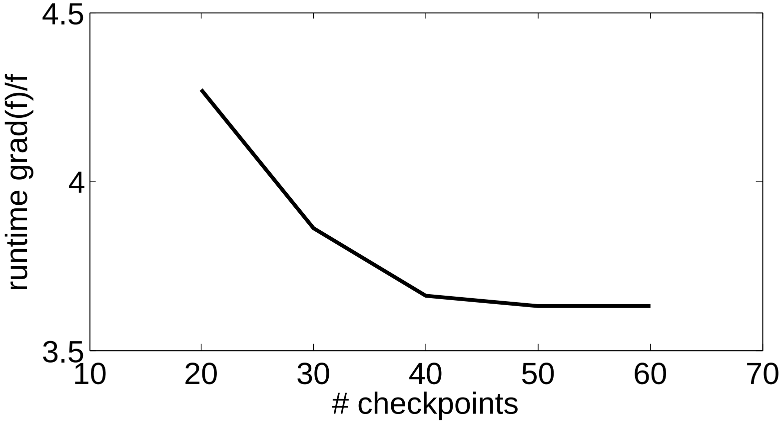
➤ acceptable level of divergence errors

execution time of one time step: 6.13s

execution time of one adjoint time step: 7.92s

➤ ratio: 1.3

Comparison of flow simulation runtime with adjoint calculation with 500 steps for 20, 30, 40, 50 and 60 checkpoints:



Outlook

Goal: adjoint-based control of turbulent flows
using electromagnetic fields

Applications: aerodynamics, crystal growth

Done:

- extension of a target function for optimization
- inclusion of hand coded adjoints and AD applications
- coupling with optimal checkpointing

Then:

- optimal control and experiments
- development of optimization strategies for turbulent flows