Robust Aerodynamic Design Using Shape Derivatives

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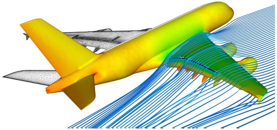
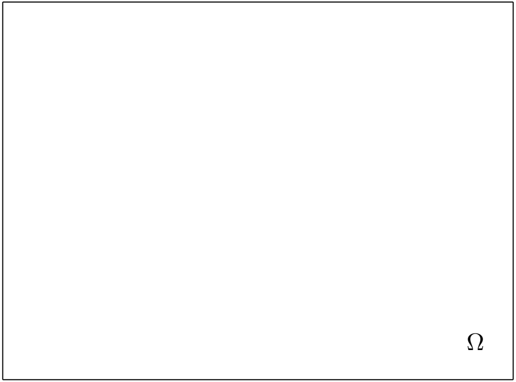


Overview

1. Modelling
2. Introduction to the Shape Calculus
3. Aerodynamic Uncertainties and Robust Optimization
4. Numerical Results

Fluid Dynamic Model



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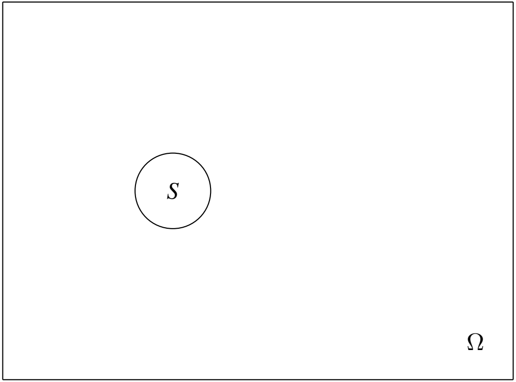

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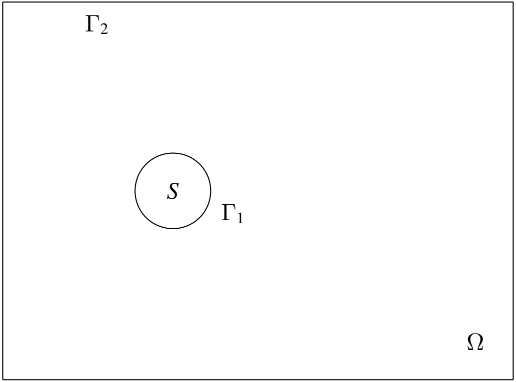

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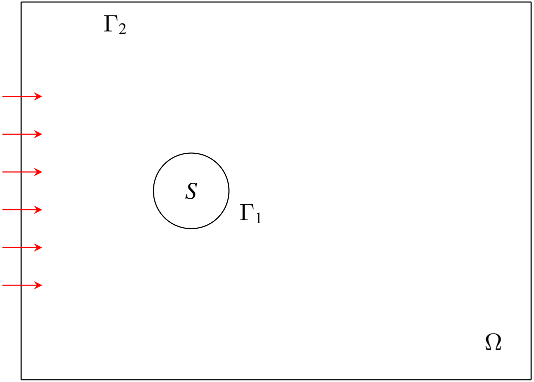

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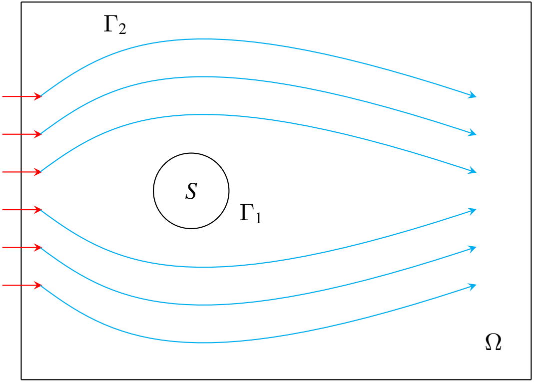

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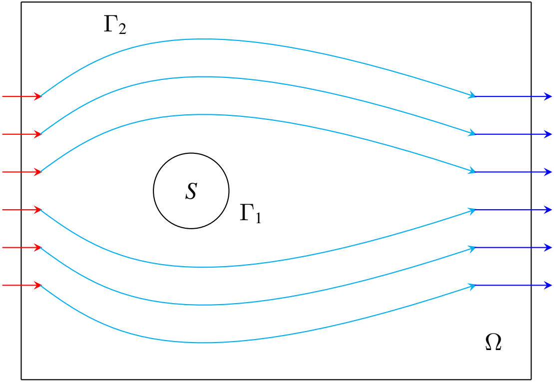
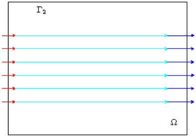


Shape Optimization Problem

problem

Find a shape of the object S so that the kinetic energy of the system is maximized with subject to constant inflow.



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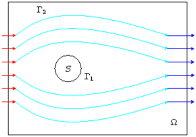

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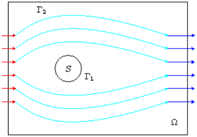

kinetic energy: $E = \frac{1}{2}m\|u\|^2 = \frac{1}{2}\int_\Omega \sum_{i=1}^n \rho u_i^2 dA$

energy dissipation: $\dot{E} = -\nu \int_\Omega \sum_{i,j=1}^n \left(\frac{\partial u_i}{\partial x_j}\right)^2 dA$

Shape Optimization Problem

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$$\min_{(u,\Omega)} J(u,\Omega) := \int_\Omega \nu \sum_{i,j=1}^{n} \left(\frac{\partial u_i}{\partial x_j} \right)^2 dA$$

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$$s.t. \quad -\nu\Delta u + u\nabla u + \nabla p = 0 \quad \text{in } \Omega$$

$$\text{div } u = 0$$

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$$\begin{aligned} s.t. \quad -\nu\Delta u + u\nabla u + \nabla p &= 0 \quad \text{in } \Omega \\ \text{div } u &= 0 \\ u &= 0 \quad \text{on } \Gamma_1 \\ u &= u_\infty \quad \text{on } \Gamma_2 \end{aligned}$$

Shape Optimization Problem

shape optimization problem

$$
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u &= u_\infty \quad \text{on } \Gamma_2 \\
h(\Omega) &\geq \text{const}_1 \quad \textit{(volume)}
\end{aligned}
$$

Shape Optimization Problem

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Introduction to the Shape Calculus (according to M.Delfour and J.Zolesio)

abstract shape optimization problem

$$\min_{\Omega} J(\Omega, u) = \int_{\Omega} F(x, u, \nabla_x u)dx$$

$$Lu = f(x) \quad \text{in } \Omega$$

$$u = g(x) \quad \text{on } \Gamma = \partial\Omega$$

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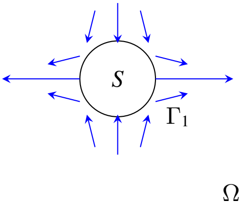
For optimization algorithm like steepest descent or Newton's method we need the derivative of the objective function $J(\Omega, u)$. Formally we get the derivative

definition: shape derivative

$$\frac{d}{d\Omega} J(\Omega) := \lim_{t \to 0} \frac{J(\Omega_t) - J(\Omega)}{t}$$

where Ω_t is a perturbation of Ω.

Introduction to the Shape Calculus (according to M.Delfour and J.Zolesio)



pertubation with velocity field V

$$\Omega_t(V) := T_t(V)(\Omega) \text{ with}$$
$$T_t : \Omega \to \mathbb{R}^n, \quad x \mapsto x + t \cdot V(x)$$

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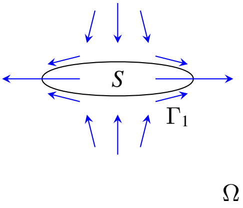

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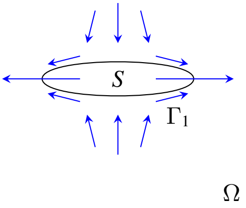

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definition: shape derivative of J in direction V

$$dJ(\Omega; V) = dJ(\Omega)[V] := \lim_{t \to 0} \frac{J(\Omega_t(V)) - J(\Omega)}{t}.$$

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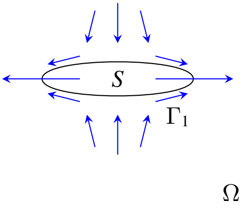

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Hadamard Structure Theorem

$$dJ(\Omega)[V] = (\nabla J, V \cdot n)_\Gamma$$

The Shape Derivative of the Shape Optimzation Problem (cf. Schulz/Schmidt

Let V be a velocity field. Then the shape derivative in direction V of the objective functional

$$J(u,\Omega) = \int_\Omega \nu \sum_{i,j=1}^{n} \left(\frac{\partial u_i}{\partial x_j}\right)^2 dA$$

is with the Green's identities

shape derivative of the shape optimization problem

$$dJ(u,\Omega)[V] = \int_{\Gamma_1} \langle V, n\rangle \left[-\nu \sum_{k=1}^{n} \left(\frac{\partial u_k}{\partial n}\right)^2 - \frac{\partial u_k}{\partial n}\frac{\partial \lambda_k}{\partial n}\right] dS.$$

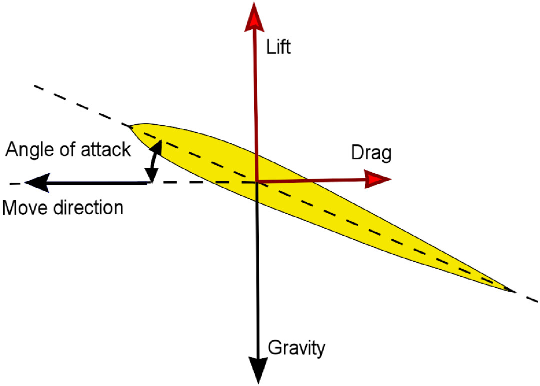
According to the Hadamard structure theorem the shape gradient is

shape gradient

$$g(x) = -\nu \sum_{k=1}^{n} \left(\frac{\partial u_k}{\partial n}(x)\right)^2 - \frac{\partial u_k}{\partial n}(x)\frac{\partial \lambda_k}{\partial n}(x), \quad x \in \Gamma_1.$$

Aerodynamic Uncertainties and Robust Optimization (cf. Schulz/Schillings)

Aerodynamic performance of an airplane is very sensitive to the wing shape and flight conditions
→ Robust design:
optimality and robustness of performance against any uncertainties.



Aerodynamic Uncertainties and Robust Optimization (cf. Schulz/Schillings)

uncertainties with respect to the flight conditions:

- Mach number
- angle of attack
- air density
- Reynolds number

Aerodynamic Uncertainties and Robust Optimization (cf. Schulz/Schillings)

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- Reynolds number

geometric uncertainties:

The uncertainties are geometric variations like buckles and dents, which have their origin in fouling, icing, characteristics of aging, manufacturing tolerance or similar.

Uncertainty modelling

single setpoint aerodynamic shape optimization problem

$$\begin{aligned} &\min_{u,q} f(u,q) \\ s.t. \quad & c(u,q) = 0 \\ & l(u,q) \geq 0. \end{aligned}$$

scalar-valued uncertainies

Let $s : \mathcal{A} \longrightarrow \mathbb{R}$ a real valued, continuous random variable defined on a probability space $(\mathcal{A}, \mathcal{Y}, \mathcal{P})$ characterized by a probability density function $\varphi : \mathbb{R} \longrightarrow \mathbb{R}_0^+$. Then the expected value can be written as

$$E(s) = \int_{\mathcal{A}} s(\zeta)\, d\mathcal{P}(\zeta) = \int_{\mathbb{R}} x\varphi(x)dx.$$

Moreover the expected value of a function $g : \mathbb{R} \longrightarrow \mathbb{R}$ is then

$$E(g(s)) = \int_{\mathcal{A}} g(\zeta) d\mathcal{P}(\zeta) = \int_{\mathbb{R}} g(x)\varphi(x)dx.$$

optimization problem influenced by stochastic pertubations

$$\begin{aligned} &\min_{u,q} f(u,q,\zeta) \\ s.t. \quad & c(u,q,\zeta) = 0 \quad \text{for all } \zeta \in \mathcal{A} \\ & l(u,q,\zeta) \geq 0 \quad \text{for all } \zeta \in \mathcal{A}. \end{aligned}$$

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robust formulations

- Min-max formulation
- Semi-infinite formulation
- Chance-constraint formulation

Semi-infinite formulation

optimization of the average objective function

$$\begin{aligned} &\min_{u_\zeta, q} \int_{\mathcal{A}} f(u_\zeta, q, \zeta) d\mathcal{P}(\zeta) \\ s.t. \quad & c(u_\zeta, q, \zeta) = 0 \quad \text{for all } \zeta \in \mathcal{A} \\ & l(u_\zeta, q, \zeta) \geq 0 \quad \text{for all } \zeta \in \mathcal{A}. \end{aligned}$$

Semi-infinite formulation

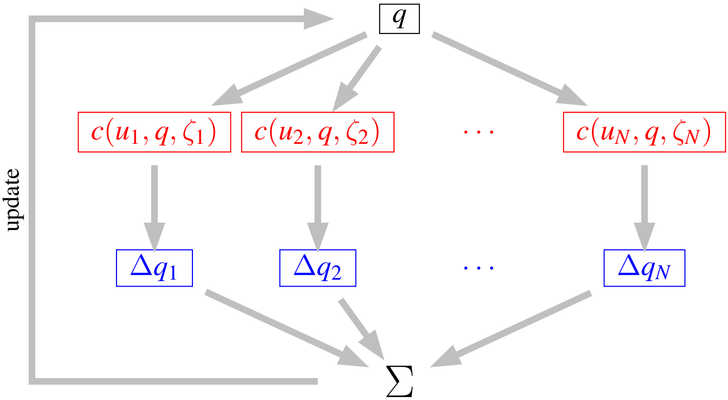
discretization leads to

$$\min_{u_i,q} \sum_{i=1}^{N} f(u_i, q, \zeta_i)\omega_i$$

$$s.t. \quad c(u_i, q, \zeta_i) = 0 \quad \text{for all } i \in \{1, \cdots, N\}$$

$$l(u_{min}, q, \zeta_{min}) \geq 0$$

Optimization Strategy



Robust Shape Optimization Problem

discretized robust shape optimization problem

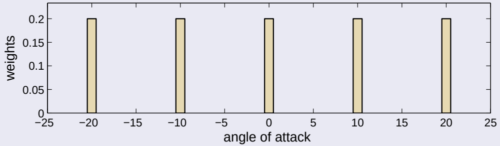
$$\min_{(u,\Omega)} J(u,\Omega,s_l) := \sum_{l=1}^{N} \int_{\Omega} \nu \sum_{i,j=1}^{2} \left(\frac{\partial u_i(s_l)}{\partial x_j} \right)^2 dA$$

$$\left.\begin{aligned} s.t. \quad -\nu\Delta u(s_l) + u(s_l)\nabla u(s_l) + \nabla p(s_l) &= 0 \quad \text{in } \Omega, \\ \operatorname{div} u(s_l) &= 0, \\ u(s_l) &= 0 \quad \text{on } \Gamma_1, \\ u(s_l) &= u_\infty \quad \text{on } \Gamma_2, \\ volume: \quad h(\Omega) &\geq \text{const}, \end{aligned}\right\} \quad \forall l = 1,\cdots,N.$$

Numerical Results - Uncertainty: angle of attack

uniform distribution

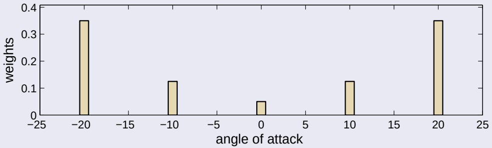
angle α_i	$-20°$	$-10°$	$0°$	$10°$	$20°$
weight ω_i	0.2	0.2	0.2	0.2	0.2



Numerical Results - Uncertainty: angle of attack

non-uniform distribution

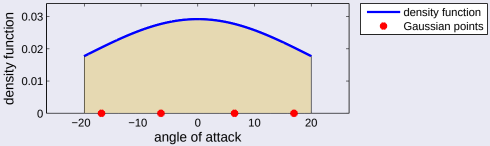
angle α_i	$-20°$	$-10°$	$0°$	$10°$	$20°$
weight ω_i	0.35	0.125	0.05	0.125	0.35



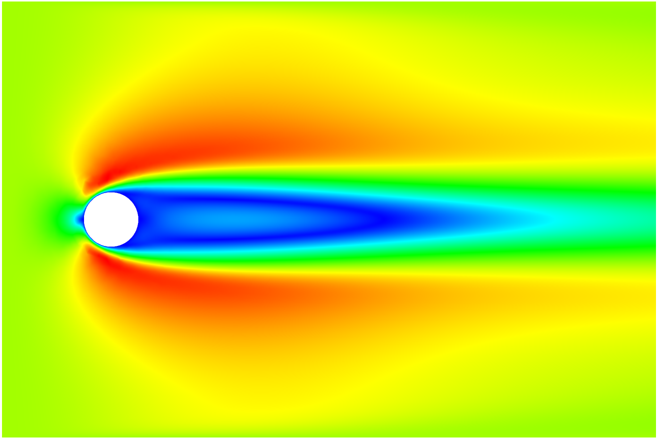
Numerical Results - Uncertainty: angle of attack

truncated normal distribution

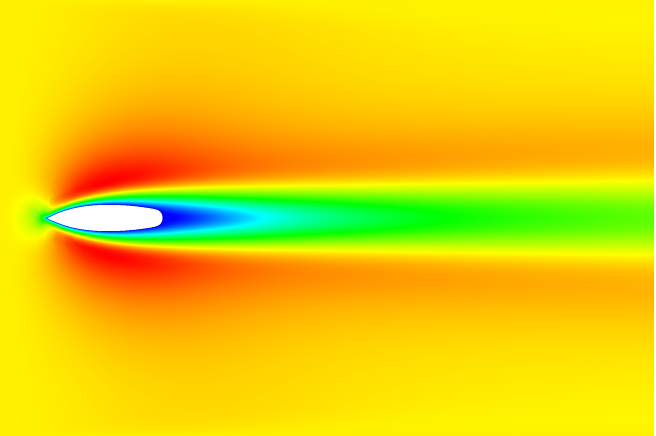
angle α_i	$-16.9635°$	$-6.4783°$	$6.4783°$	$16.9635°$
weight ω_i	0.1515	0.3485	0.3485	0.1515



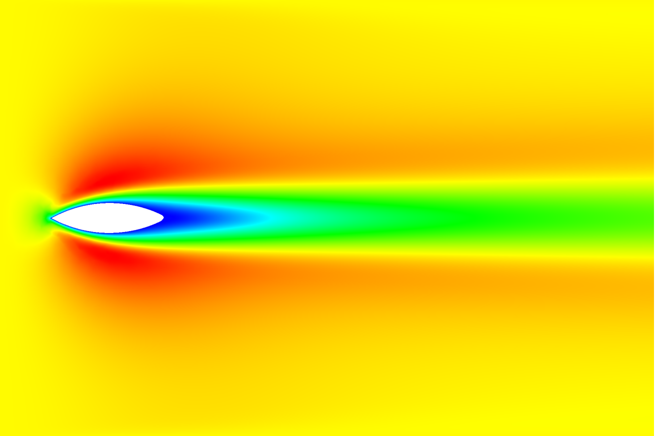
Init case: circle (Fluid-Model: Navier-Stokes-Equations)



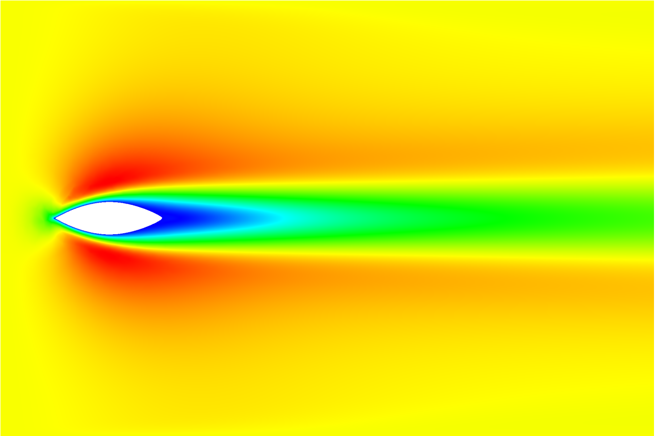
single-set



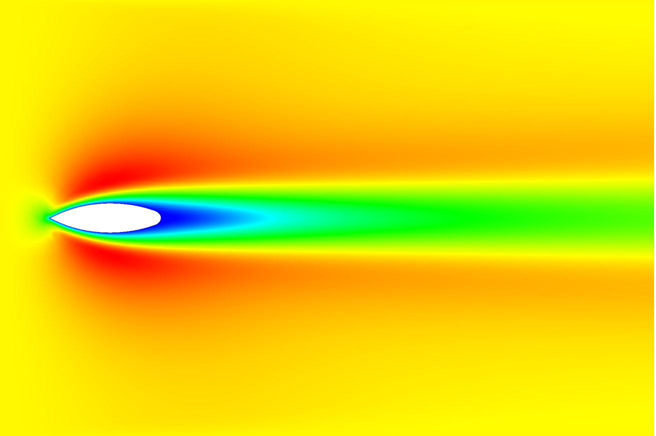
uniform



non-uniform

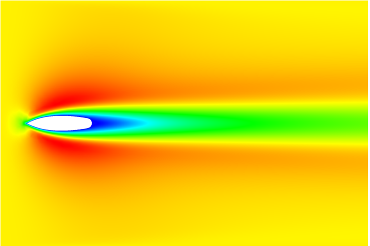


truncated normal

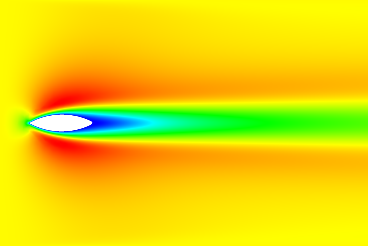


Robust Shapes

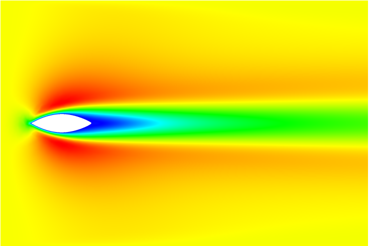
single-set



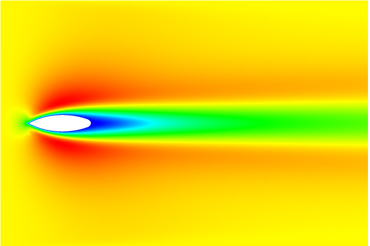
uniform



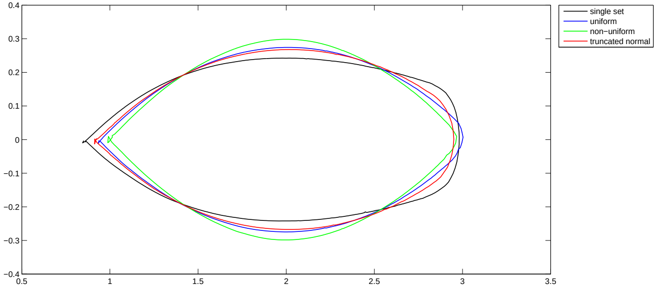
non-uniform



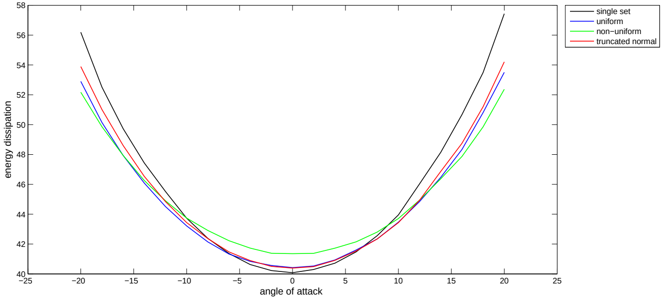
truncated normal



Comparison


0.4
0.3
0.2
0.1
0
−0.1
−0.2
−0.3
−0.4
0.5
1
1.5
2
2.5
3
3.5
single set
uniform
non−uniform
truncated normal

Comparison


single set
uniform
non−uniform
truncated normal
energy dissipation
angle of attack
58
56
54
52
50
48
46
44
42
40
−25
−20
−15
−10
−5
0
5
10
15
20
25

Comparison

shape	integral	energy diss. at 0°	energy diss at 20°
circle	1.419495	1.595814	1.118724
single set	1.0000	1.0000	1.0000
uniform	0.980115	1.008454	0.931625
non-uniform	0.986333	1.028772	0.911846
truncated normal	0.985439	1.006942	0.943860

Conclusion and Future Work

conclusion:

- Shape calculus is a powerful tool to compute the shape gradient
- Semi-infinte formulation is a good choice to model uncertainties in the robust optimization
- Combination of both techniques leads to good results in robust aerodynamic design

further work:

- Parallelization
- Coupling of shape and structural optimization