J. OPERATOR THEORY
87:1(2022), 25–40

DYNAMICS OF THE TAYLOR SHIFT ON BERGMAN SPACES

JÜRGEN MÜLLER and MAIKE THELEN

Communicated by Hari Bercovici

ABSTRACT. The Taylor (backward) shift on Bergman spaces $A^p(\Omega)$ for general open sets Ω in the extended complex plane shows rich variety concerning its dynamical behaviour. Different aspects are worked out, where in the case $p < 2$ a recent result of Bayart and Matheron plays a central role.

KEYWORDS: *Mixing, metric dynamics, topological dynamics*

MSC (2010): 47A16, 47A35

1. INTRODUCTION

Let Ω be an open subset of the Riemann sphere $\mathbb{C}_\infty$, where $\mathbb{C}_\infty$ is equipped with the spherical metric. Moreover, let $H(\Omega)$ denote the Fréchet space of functions holomorphic in Ω and vanishing at ∞, endowed with the topology of compact convergence. If $0 \in \Omega$, the Taylor (backward) shift $T : H(\Omega) \to H(\Omega)$ is defined by

$$(Tf)(z) := \begin{cases} (f(z) - f(0))/z, & z \neq 0 \\ f'(0), & z = 0 \end{cases}.$$

It is easily seen that T is a continuous operator on $H(\Omega)$. Moreover, the n-th iterate T^n is given by

$$(T^n f)(z) := \begin{cases} (f - S_{n-1}f)(z)/z^n, & z \neq 0 \\ a_n, & z = 0 \end{cases},$$

where $(S_n f)(z) := \sum_{\nu=0}^{n} a_\nu z^\nu$ denotes the n-th partial sum of the Taylor expansion of f about 0. In particular, for $|z| < \operatorname{dist}(0, \partial\Omega)$ we have

$$(T^n f)(z) = \sum_{\nu=0}^{\infty} a_{\nu+n} z^\nu ,$$

that is, locally at 0 the Taylor shift acts as backward shift on the Taylor coefficients.

An important feature of the Taylor shift is that the spectrum is easily determined: Setting $A^* = 1/(\mathbb{C}_\infty \setminus A)$ for $A \subset \mathbb{C}_\infty$, the set Ω^* is compact in the complex plane $\mathbb{C}$ if and only if Ω is open in $\mathbb{C}_\infty$ with $0 \in \Omega$. For $\alpha \in \mathbb{C}$, we define $\gamma(\alpha)\colon \{\alpha\}^* \to \mathbb{C}$ by

$$\gamma(\alpha)(z) := \frac{1}{1-\alpha z} \quad (z \in \mathbb{C}_\infty \setminus \{1/\alpha\}). \tag{1.1}$$

Since $\gamma(\alpha) \in H(\Omega)$ is an eigenfunction to the eigenvalue α for all $\alpha \in \Omega^*$, the point spectrum contains Ω^*. Moreover, the corresponding eigenspace is one-dimensional. On the other hand, a calculation shows that for $1/\alpha \in \Omega$ the operator $S_\alpha : H(\Omega) \to H(\Omega)$ defined by

$$(S_\alpha g)(z) = \frac{zg(z) - g(1/\alpha)/\alpha}{1 - z\alpha} \qquad (z \in \Omega \setminus \{1/\alpha\})$$

(and appropriately extended at $1/\alpha$) is the continuous inverse to $T - \alpha I$ and hence the spectrum and the point spectrum both equal Ω^*.

The Taylor shift may also be considered as an operator on Banach spaces of functions holomorphic in Ω as e.g. Bergman spaces, that is, subspaces of $H(\Omega)$ of functions which are p-integrable with respect to the two-dimensional Lebesgue measure. In the case of the open unit disc $\Omega = \mathbb{D}$ there is an elaborated theory about invariant subspaces and cyclic vectors for Hardy- and Bergman spaces (see e.g. [7], cf. also [10]). Since we are interested also – and in particular – in the case of open sets Ω containing ∞ and in order to avoid difficulties according to local integrability at ∞, we modify the usual Bergman spaces and consider the surface measure on the sphere $\mathbb{C}_\infty$ instead. We denote the normalized surface measure by m_2 and, correspondingly, the normalized arc length measure on the unit circle $\mathbb{T}$ by m_1 of briefly m.

For $1 \le p < \infty$ and $\Omega \subset \mathbb{C}_\infty$ open we define the Bergman space $A^p(\Omega) = A^p(\Omega, m_2)$ as the set of all functions $f \in H(\Omega)$ which fulfil

$$\|f\|_p := \left(\int_\Omega |f|^p \, dm_2 \right)^{1/p} < \infty .$$

Then $(A^p(\Omega), \|\cdot\|_p)$ is a Banach space. If Ω is open and bounded in $\mathbb{C}$, the above norm and the classical p-norm with respect to Lebesgue measure are equivalent.

In case $0 \in \Omega$, the Taylor shift turns out to be a continuous operator on $A^p(\Omega)$. For $\alpha \in (\Omega^*)^\circ$ the functions $\gamma(\alpha)$ belong to $A^p(\Omega)$ for all p and it is clear that $\gamma(\alpha)$ is an eigenfunction to the eigenvalue α. Again, for $1/\alpha \in \Omega$, the operator S_α from above, now defined on $A^p(\Omega)$, turns out to be the continuous inverse to $T - \alpha I$. Moreover, in the case $p < 2$ the functions $\gamma(\alpha)$ belong to $A^p(\Omega)$ also for $\alpha \in \partial\Omega^*$, which yields that in this case the point spectrum equals Ω^*. Thus, we obtain:

(i) $(\Omega^*)^\circ \subset \sigma_0(T)$ and $\overline{(\Omega^*)^\circ} \subset \sigma(T) \subset \Omega^*$ for all $p \ge 1$.
(ii) $\sigma_0(T) = \sigma(T) = \Omega^*$ for $1 \le p < 2$.

This gives high flexibility in prescribing spectra. In particular, each compact plane set K appears as spectrum and point spectrum of T on $A^p(K^*)$ for $1 \le p < 2$. For $p \ge 2$ the situation is more delicate. In general $\gamma(\alpha)$ does not belong to

$A^p(\Omega)$ for $\alpha \in \partial\Omega^*$. If, however, Ω is "sufficiently small" near a boundary point $1/\alpha$ of Ω, it may happen that $\gamma(\alpha)$ does belong to $A^p(\Omega)$. A simple example is the crescent-shaped region $\Omega = \mathbb{D} \setminus \{z : |z - 1/2| \leq 1/2\}$, where $\gamma(1) \in A^2(\Omega)$. This opens the possibility to choose Ω in such a way that eigenvalues are placed at certain points.

In [4], [5] and [23], the behaviour of the Taylor shift with respect to topological dynamics was studied. We recall that an operator T on a separable Fréchet space X is called *hypercyclic* if T has a dense orbit. This is equivalent to T being *topologically transitive*, that is, for any two nonempty open sets $U, V \subset X$ the images $T^n(U)$ meet V infinitely often. Moreover, T is *topologically mixing* if $T^n(U)$ meets V for all sufficiently large n. Concerning these and further notions from topological (linear) dynamics we refer the reader to [2] and [13].

The main result from [4] states that the following are equivalent:

- T is topologically mixing
- T is hypercyclic
- Each component of Ω^* meets the unit circle $\mathbb{T}$.

The situation changes drastically if we consider Bergman spaces. If T is hypercyclic on $A^p(\Omega)$, for some $p < 2$, then Ω^* has to be perfect. In [5] it is shown that T is mixing on $A^p(\Omega)$ if $\Omega \supset \mathbb{D}$ is a Jordan domain such that $\Omega^* \cap \mathbb{T}$ contains an arc.

In Section 2 we study the Taylor shift operator for its metric dynamical properties. For $H(\Omega)$ and in the case of $A^p(\Omega)$ with $p < 2$, the sufficient supply of eigenvectors $\gamma(\alpha)$ allows the application of a recent deep result of Bayart and Matheron (Theorem 1.1 from [3]) which in many respects finishes a line of investigations concerning relations between the existence of unimodular eigenvectors and the dynamics of a linear operator.

This is no longer possible for $p \geq 2$. In Section 3 we show that the Taylor shift is topologically mixing on $A^p(\Omega)$ for arbitrary p if each component of Ω^* is sufficiently large near the unit circle $\mathbb{T}$.

2. METRIC DYNAMICS OF T

In this section, we investigate the Taylor shift on $H(\Omega)$ and $A^p(\Omega)$ for $p < 2$ with respect to its metric dynamical behaviour. We recall that a measure-preserving transformation T on a probability space (X, Σ, μ) is called *weakly mixing (with respect to μ)* if for any $A, B \in \Sigma$

$$\lim_{N \to \infty} \frac{1}{N} \sum_{n=0}^{N-1} |\mu(A \cap T^{-n}(B)) - \mu(A)\mu(B)| = 0$$

and it is called *strongly mixing (with respect to μ)* if for any $A, B \in \Sigma$

$$\mu(A \cap T^{-n}(B)) \to \mu(A)\mu(B) \quad (n \to \infty).$$

For further notions from ergodic theory we refer the reader e.g. to [24]. Consider now X to be a complex separable Fréchet space. Each operator T which is weakly mixing with respect to some measure of full support is frequently hypercyclic (see e.g. [2, Corollary 5.5]) and then also hypercyclic. Moreover, strong mixing with respect to some measure of full support implies topological mixing.

The operator T is called weakly (resp. strongly) mixing *in the Gaussian sense* if it is weakly (resp. strongly) mixing with respect to some Gaussian probability measure μ having full support. The definition of Gaussian probability measures and related results can be found in [2] and [3]. For the notion of the cotype of a Banach space we refer to [1].

Let now T be the Taylor shift on $H(\Omega)$ or $A^p(\Omega)$, where $1 \leq p < 2$. In order to treat both cases simultaneously, we write $A^0(\Omega) := H(\Omega)$. Then, for $D \subset \mathbb{T}$,

$$\operatorname{span} \bigcup_{\alpha \in \mathbb{T} \setminus D} \ker(T - \alpha I) = \operatorname{span}\big(\gamma(\Omega^* \cap \mathbb{T} \setminus D)\big).$$

For $\Lambda \subset \mathbb{T}$ we say that $\gamma(\Lambda)$ is *perfectly spanning* in $A^p(\Omega)$ if the span of $\gamma(\Lambda \setminus D)$ is dense in $A^p(\Omega)$ for all countable $D \subset \mathbb{T}$. Similarly, we say that $\gamma(\Lambda)$ is $\mathcal{U}_0$-*perfectly spanning* if this holds for all $D \in \mathcal{U}_0$, where $\mathcal{U}_0$ denotes class of sets of extended uniqueness (see e.g. [16, p. 76]). We recall that all sets of extended uniqueness have vanishing arc length measure.

Since for $1 \leq p \leq 2$ the Bergman space $A^p(\Omega)$ as closed subspace of $L^2(\Omega, m_2)$ is of cotype 2, we obtain as an immediate consquence of the Bayart-Matheoren theorem mentioned in the introduction (Theorem 1.1 from [3])

THEOREM 2.1. *Let $0 \in \Omega \subset \mathbb{C}_\infty$ be an open set and let T be the Taylor shift on $A^p(\Omega)$, where $p \in \{0\} \cup [1, 2)$.*

(i) *If $\gamma(\Omega^* \cap \mathbb{T})$ is perfectly spanning in $A^p(\Omega)$ then T is weakly mixing in the Gaussian sense.*
(ii) *If $\gamma(\Omega^* \cap \mathbb{T})$ is $\mathcal{U}_0$-perfectly spanning in $A^p(\Omega)$ then T is strongly mixing in the Gaussian sense.*

If $p \in [1, 2)$ then in both cases the converse implication is true.

We say that a point $z \in \mathbb{C}$ is a *perfect limit point* of $A \subset \mathbb{C}$ if $U \cap A$ is uncountable for each neighbourhood U of z, that is, if z is a limit point of $A \cap U \setminus D$ for each countable set D. Similarly, we say that z is a $\mathcal{U}_0$-*perfect limit point* if z is a limit point of $A \cap U \setminus D$ for each neighbourhood U of z and each $D \in \mathcal{U}_0$. If $A \subset \mathbb{T}$ has locally positive arc length measure at z then z is a $\mathcal{U}_0$-perfect limit point. Applying an appropriate version of Runge's theorem which can be found e.g. in [17, Theorem 10.2] we obtain from Theorem 2.1:

COROLLARY 2.2. *Let $0 \in \Omega \subset \mathbb{C}_\infty$ be an open set and let T be the Taylor shift on $H(\Omega)$.*

(i) *If each component of* Ω^* *contains a perfect limit point of* $\Omega^* \cap \mathbb{T}$, *then* T *is weakly mixing in the Gaussian sense.*

(ii) *If each component of* Ω^* *contains a* $\mathcal{U}_0$*-perfect limit point of* $\Omega^* \cap \mathbb{T}$, *then* T *is strongly mixing in the Gaussian sense.*

REMARK 2.3. By separating singularities it is easily seen from [23, Corollary 1] that Ω^* necessarily has to be perfect if the Taylor shift on $H(\Omega)$ is weakly mixing (or merely frequently hypercyclic). Note that $\Omega^* \cap \mathbb{T}$ not necessarily has to be perfect: If B is some closed arc on $\mathbb{T}$ symmetric to the real axis and

$$\Omega = \mathbb{C} \setminus \big(B \cup (-\infty, -1] \cup [1, \infty)\big)$$

then $\Omega^* = B \cup [-1, 1]$ satisfies the assumption of Corollary 2.2.2, hence T is strongly mixing on $H(\Omega)$.

We turn to Bergman spaces. Theorem 2.1 shows that the question whether T is (strongly or weakly) mixing completely reduces to a question about mean approximation by rational functions with simple poles in appropriate subsets of $\mathbb{T}$. The following result on separation of singularities implies that the general question may be reduced to special cases.

PROPOSITION 2.4. *Let* $p \geq 1$ *and let* $\Omega_1, \Omega_2 \subset \mathbb{C}_\infty$ *be open sets in* $\mathbb{C}_\infty$ *with* $\Omega_1 \cup \Omega_2 = \mathbb{C}_\infty$. *Then* $A^p(\Omega_1 \cap \Omega_2) = A^p(\Omega_1) \oplus A^p(\Omega_2)$.

Proof. It is known that, by separation of singularities of holomorphic functions,

$$H(\Omega_1 \cap \Omega_2) = H(\Omega_1) \oplus H(\Omega_2)$$

as topological direct sum. Since convergence in $A^p(\Omega)$ implies convergence in $H(\Omega)$ (see e.g. [9, Chapter 1, Theorem 1]), it suffices to show that for $f \in A^p(\Omega_1 \cap \Omega_2)$ decomposed as $f = f_1 + f_2 \in H(\Omega_1) \oplus H(\Omega_2)$ we have $f_j \in A^p(\Omega_j)$ for $j = 1, 2$.

Let $f \in A^p(\Omega_1 \cap \Omega_2)$ and $f = f_1 + f_2$ with $f_j \in H(\Omega_j)$. Since the boundary of $\Omega_1 \cap \Omega_2$ is the union of the (compact) boundaries $\partial\Omega_j \subset \Omega_{3-j}$, for $j = 1, 2$, we can find compact disjoint neighbourhoods $U_j \subset \Omega_{3-j}$ of $\partial\Omega_j$. Then

$$\int_{\Omega_j} |f_j|^p dm_2 = \int_{\Omega_j \setminus U_j} |f_j|^p dm_2 + \int_{\Omega_j \cap U_j} |f - f_{3-j}|^p dm_2 < \infty.$$

This yields $f_j \in A^p(\Omega_j)$ for $j = 1, 2$. ∎

An immediate consequence is the fact that hypercyclicity of T on $A^p(\Omega)$, for some $1 \leq p < 2$, implies that Ω^* is perfect: Suppose that ζ is an isolated point of Ω^*. Then we have

$$A^p(\Omega) = A^p(\Omega \cup \{1/\zeta\}) \oplus A^p(\mathbb{C}_\infty \setminus \{1/\zeta\}).$$

By [13, Proposition 2.25] it follows that T is also hypercyclic on $A^p(\mathbb{C}_\infty \setminus \{1/\zeta\})$. Since $A^p(\mathbb{C}_\infty \setminus \{1/\zeta\})$ reduces to the span of $\gamma(\zeta)$ and is thus one-dimensional we get a contradiction.

In order to be able to reduce the case of open sets Ω containing ∞ to the case of bounded open sets in $\mathbb{C}$ we recall

PROPOSITION 2.5. *Let X a Fréchet space and let L be complemented in X. If $X = L \oplus M$ and if $A \subset L$ and $B \subset M$ with* $\operatorname{span}(A + B)$ *dense in X then* $\operatorname{span}(A)$ *is dense in L.*

Proof. Let $a \in L$. Then a sequence (x_n) in $\operatorname{span}(A + B)$ exists with $x_n \to a$ in X as n tends to ∞. We write $x_n = a_n + b_n$ with $a_n \in \operatorname{span} A$ and $b_n \in \operatorname{span} B$. Since a belongs to L and since the projection of X to L along M is continuous (see e.g. [21, Theorem 5.16]), the sequence (a_n) converges to a. ∎

REMARK 2.6. Let Ω be open with $\infty \in \Omega$ and $\rho > \max_{z\in\mathbb{C}_\infty\setminus\Omega} |z|$. If we put $\Omega_\rho := \Omega \cap \rho\mathbb{D}$ then

$$A^p(\Omega_\rho) = A^p(\Omega) \oplus A^p(\rho\mathbb{D})$$

and $\gamma(\rho^{-1}\mathbb{D}) \subset A^p(\rho\mathbb{D})$ for all $p \geq 1$. If $B \subset A^p(\Omega)$ is so that $B + \gamma(\rho^{-1}\mathbb{D})$ densely spans $A^p(\Omega_\rho)$ then B has dense span in $A^p(\Omega)$ by Proposition 2.4.

For $K \subset \mathbb{C}$ compact, a set $\Lambda \subset K$ is called a *K-uniqueness set* if every continuous function on K which is holomorphic in the interior of K and vanishes on Λ vanishes identically. Obviously, if K is nowhere dense then Λ is a K-uniqueness set if and only if Λ is dense in K. More generally, it is easily seen that $\Lambda \subset K$ is a K-uniqueness set if and only if $K \setminus \overline{K^\circ} \subset \overline{\Lambda}$ and for every component C of K° the set $\Lambda \cap \overline{C}$ is a uniqueness set for $\overline{C}$.

With that notion, we have the following result on rational approximation. For the case $p = 1$ and Lebesgue measure instead of surface measure the result is due to Bers ([6]).

THEOREM 2.7. *Let $1 \leq p < 2$ and $0 \in \Omega \subset \mathbb{C}_\infty$ be an open set which is either bounded in $\mathbb{C}$ or contains ∞. Moreover, suppose Λ to be a subset of Ω^*.*

(i) *If Λ is a Ω^*-uniqueness set then the span of $\gamma(\Lambda)$ is dense in $A^p(\Omega)$.*
(ii) *If $m_2(\Omega^*) = 0$ then the span of $\gamma(\Lambda)$ is dense in $A^p(\Omega)$ if and only if Λ is dense in Ω^*.*

Proof. 1. We first assume that Ω is bounded in $\mathbb{C}$. Let $\ell \in A^p(\Omega)'$ with $\ell(\gamma(\alpha)) = 0$ for all $\alpha \in \Lambda$. Since $A^p(\Omega)$ is a subspace of $L^p(\Omega)$ the Hahn-Banach theorem yields that ℓ can be extended to a continuous linear functional on $L^p(\Omega)$. Thus, there exists a function $g \in L^q(\Omega)$, where q is the conjugated exponent, such that

$$\ell(f) = \int_\Omega f\overline{g}\, dm_2$$

for all $f \in A^p(\Omega)$. For the measure $1_\Omega g dm_2 \in M(\overline{\Omega})$ the Cauchy transform

$$(Vg)(\alpha) := \int_\Omega \frac{\overline{g}(\zeta)}{1 - \zeta\alpha} dm_2(\zeta) = \frac{1}{\alpha} \int_\Omega \frac{\overline{g}(\zeta)}{1/\alpha - \zeta} dm_2(\zeta)$$

of $1_\Omega g dm_2$ is holomorphic in the interior of Ω^* and continuous in $\mathbb{C}$ as the convolution of $w \mapsto 1/w \in A_p(\mathbb{C}_\infty \setminus \{0\})$ and the function $1_\Omega g \in L^q(\mathbb{C})$. Since

$$(Vg)(\alpha) = \ell(\gamma(\alpha)) = 0$$

for all $\alpha \in \Lambda$ and since Λ is a Ω^*-uniqueness set we have that $Vg|_{\Omega^*} = 0$ and thus $\ell(\gamma(\alpha)) = 0$ for all $\alpha \in \Omega^*$. So ℓ vanishes on the set of rational functions with simple poles in $\mathbb{C} \setminus \Omega$. According to (the proof of) [14, Theorem 1], the set of these functions is dense in $A^p(\Omega)$. This yields that $\ell = 0$ and then the Hahn-Banach theorem implies the assertion.

Now, let Ω be open with $\infty \in \Omega$ and Ω_ρ as in Remark 2.6. Then $\Omega_\rho^* = \Omega^* \cup \rho^{-1}\overline{\mathbb{D}}$. Since $\Lambda \cup \rho^{-1}\mathbb{D}$ is a Ω_ρ^*-uniqueness set, by the previous considerations we have that the span of $\gamma(\Lambda \cup \rho^{-1}\mathbb{D})$ is dense in $A^p(\Omega_\rho)$. By Remark 2.6 the span of $\gamma(\Lambda)$ is dense in $A^p(\Omega)$.

2. It is easily seen that in case $m_2(\Omega^*) = 0$ the denseness of Λ in Ω^* is necessary for $\gamma(\Lambda)$ to be densely spanning in $A^p(\Omega)$. Conversely, since Ω^* is nowhere dense, denseness of Λ in Ω^* implies Ω^*-uniqueness. ∎

If Ω^* has interior points then Ω^*-uniqueness of Λ is in general not necessary for $\gamma(\Lambda)$ to be (even perfectly) spanning in $A^p(\Omega)$:

EXAMPLE 2.8. Let $0 < \delta < 1$ and $E_\delta := (1 + iC_\delta)$, where C_δ is the convex hull of the closed curve bounded by $\{t + i\varphi(t) : -\delta \le t \le \delta\}$ with

$$\varphi(t) := e^{-1/|t|} + 1 - \sqrt{1 - t^2} \quad (-\delta \le t \le \delta)$$

(where $e^{-\infty} := 0$) and the horizontal line $\{t + i\varphi(\delta) : -\delta \le t \le \delta\}$. Since each dense subset of $\mathbb{T}$ is a $\mathbb{D}^*$-uniqueness set, $\gamma(\mathbb{T})$ is $\mathcal{U}_0$-perfectly spanning in $A^p(\mathbb{D})$ for $p < 2$. For the crescent-shaped domain $\Omega := \mathbb{D} \setminus E_\delta$, however, $\mathbb{T}$ is no Ω^*-uniquensess set. On the other hand, the domain Ω is so "sharp" near the point 1 that the polynomials form a dense subspace of $A^2(\Omega)$ (see Theorem 12.1 in [18]; cf. also [11, p. 29]) and thus of $A^p(\Omega)$ for $p < 2$. In particular, $A^p(\mathbb{D})$ is dense in $A^p(\Omega)$. But then $\gamma(\mathbb{T})$ is also $\mathcal{U}_0$-perfectly spanning in $A^p(\Omega)$ for $p < 2$ and, according to Theorem 2.1, the Taylor shift on $A^p(\Omega)$ is strongly mixing in the Gaussian sense.

From the second part of Theorem 2.7 we obtain a quite complete characterization of the metric dynamics of T for the case of open sets Ω with $m_2(\Omega^*) = 0$. We recall that for any perfect set $A \subset \mathbb{T}$ each point in A is a perfect limit point. A closed set $A \subset \mathbb{T}$ is said to be *$\mathcal{U}_0$-perfect* if $U \cap A \notin \mathcal{U}_0$ for all open sets U that meet A. In particular, closed sets $A \subset \mathbb{T}$ which have locally positive arc length measure are $\mathcal{U}_0$-perfect. For any $\mathcal{U}_0$-perfect set $A \subset \mathbb{T}$ each point in A is a $\mathcal{U}_0$-perfect limit point.

THEOREM 2.9. *Let $\Omega \subset \mathbb{C}_\infty$ be open with $0, \infty \in \Omega$ and $m_2(\Omega^*) = 0$. Furthermore, let $1 \le p < 2$ and T be the Taylor shift on $A^p(\Omega)$.*

(i) *T is weakly mixing in the Gaussian sense if and only if Ω^* is a perfect subset of $\mathbb{T}$,*
(ii) *T is strongly mixing in the Gaussian sense if and only if Ω^* is a $\mathcal{U}_0$-perfect subset of $\mathbb{T}$.*

Proof. If $\Omega^* \subset \mathbb{T}$ is perfect, then $\Omega^* \setminus D$ is dense in Ω^* for all countable sets D. Theorem 2.1 and Theorem 2.7 show that T is weakly mixing in the Gaussian sense. In the same way, Theorem 2.1 and Theorem 2.7 show that T is strongly mixing in the Gaussian sense if $\Omega^* \subset \mathbb{T}$ is $\mathcal{U}_0$-perfect.

On the other hand, as noted above, already hypercylicity of T requires perfectness of Ω^*. Since $m_2(\Omega^*) = 0$, Theorem 2.1 and Theorem 2.7 show that the set Ω^* has to be a subset of $\mathbb{T}$. Moreover,

$$\Omega^* \ni \alpha \to \gamma(\alpha) \in A^p(\Omega)$$

defines a continuous eigenvector field for T. The same arguments as in Example 2 of [3] show that $\mathcal{U}_0$-perfectness of $\Omega^* \cap \mathbb{T}$ is necessary for T to be strongly mixing on $A^p(\Omega)$ for any $1 \le p < 2$. ∎

EXAMPLE 2.10. Theorem 2.9 implies that for each set $B \subset \mathbb{T}$ which has locally positive arc length measure (as e.g. a nontrivial arc) the Taylor shift T on $A^p(\mathbb{C}_\infty \setminus B)$ is strongly mixing in the Gaussian sense for $p < 2$. If B is perfect but not $\mathcal{U}_0$-perfect then T is weakly mixing but not strongly mixing in the Gaussian sense.

For the case that Ω^* has interior points we can show

THEOREM 2.11. *Let $0 \in \Omega \subset \mathbb{C}_\infty$ be an open set which is either bounded in $\mathbb{C}$ or contains ∞. If each component K of Ω^* is the closure of a simply connected domain G such that the harmonic measure $\omega(\cdot, K \cap \mathbb{T}, G)$ is positive or G meets $\mathbb{T}$ then the Taylor shift on $A^p(\Omega)$ is strongly mixing in the Gaussian sense for all $p < 2$.*

Proof. From the two-constant-theorem (see e.g. [20]) it follows that for each domain G with non polar boundary sets $A \subset \partial G$ of positive harmonic measure $\omega(\cdot, A, G)$ are uniqueness sets for G. If K is a component of Ω^* then, according to our assumptions, the local F. and M. Riesz theorem (see [12, p. 415]) shows that $m(K \cap \mathbb{T})$ is positive. Since each $\mathcal{U}_0$-set $D \subset \Omega^* \cap \mathbb{T}$ has vanishing arc length measure and, again by the local F. and M. Riesz theorem, also vanishing harmonic measure, $(\Omega^* \cap \mathbb{T}) \setminus D$ is a Ω^*-uniqueness set for all $D \in \mathcal{U}_0$. Hence, according to Theorem 2.1 and Theorem 2.7 the Taylor shift on $A^p(\Omega)$ is strongly mixing in the Gaussian sense for all $p < 2$. ∎

REMARK 2.12. Let Ω with $0 \in \Omega$ be the exterior of a rectifiable Jordan curve Γ. Then the interior $G = (\Omega^*)^\circ$ of $1/\Gamma$ is a Jordan domain with rectifiable boundary and, according to the (global) F. and M. Riesz theorem (see e.g. [12, p. 202]), the harmonic measure of a set $A \subset \partial G$ is positive if and only if the linear measure is positive. For $A \subset \mathbb{T}$ this in turn is equivalent to A having positive arc length measure. Hence, if $m(\Omega^* \cap \mathbb{T}) > 0$, then T is strongly mixing in the Gaussian sense for all $p < 2$.

Let λ_2 denote the two-dimensional Lebesgue measure and let

$$D_q(G) := \{h \in H(G) : \int_G |h'|^q \, d\lambda_2 < \infty\}$$

be the Dirichlet space of order q with respect to G. In a similar way as in the proof of Theorem 1 in [19], by applying Theorem 3, Chapter II, Section 4, from [22], it can be shown that for Cauchy transforms Vg of functions $g \in L^q(\Omega)$ as considered in the proof of Theorem 2.7 the restrictions $Vg|_G$ belong to $D_q(G)$ and thus in particular to $D_2(G)$. It is known that for the Dirichlet space $D_2(\mathbb{D})$ perfect uniqueness sets of vanishing arc length measure exist (see [10, Corollary 4.3.4]). By conformal invariance (cf. [10, Therorem 1.4.1]), the rectifiable Jordan curve Γ can be chosen in such a way that $m(\Omega^* \cap \mathbb{T}) = 0$ and that T is weakly mixing in the Gaussian sense for all $p < 2$.

EXAMPLE 2.13. Let $C \subset \mathbb{T}$ be a closed set and consider Γ to be a rectifiable Jordan curve in $\mathbb{C} \setminus \mathbb{D}$ with $\Gamma \cap \mathbb{T} = C$ and so that the exterior Ω of Γ contains 0. If C has positive arc length measure then Remark 2.12 shows that the Taylor shift T on $A^p(\Omega)$ is strongly mixing in the Gaussian sense for $p < 2$. Note that C may be chosen to be a totally disconnected set and that C may have isolated points. Moreover, according to the proof of [10, Corollary 4.3.4], for an appropriate countable union C of circular Cantor middle-third sets the Taylor shift weakly mixing in the Gaussian sense for $p < 2$.

3. TOPOLOGICAL DYNAMICS OF T

If Ω is an open set such that no point of $\mathbb{T}$ is an interior point of Ω and if $p \geq 2$, the Taylor shift T on $A^p(\Omega)$ may have no unimodular eigenvalues. This is e.g. the case for $\Omega = \mathbb{D}$. Since $A^2(\Omega)$ is of cotype 2, Theorem 2.1 shows that weak mixing in the Gaussian sense is excluded. We recall that an operator T on a Fréchet space X is called *frequently hypercyclic* if the orbit of some point x meets each nonempty open set with positive lower density. Each operator that is weakly mixing with respect to some measure of full support is frequently hypercyclic.

The space $A^2(\mathbb{D})$ is isometrically isomorphic to the weighted sequence space $\ell^2(1/(n+1))$ and the Taylor shift is conjugated to the backward shift on the space $\ell^2(1/(n+1))$ (see [13, Example 4.4.(b)]). As a consequence, the Taylor shift is topologically mixing but not frequently hypercyclic on $A^2(\mathbb{D})$ ([13, Example 9.18]). It turns out that a similar result holds for the Taylor shift on more general domains Ω and for arbitrary p.

THEOREM 3.1. *Let $1 \leq p < \infty$ and $0 \in \Omega \subset \mathbb{C}_\infty$ be a domain which is either bounded in $\mathbb{C}$ or contains ∞. If each component K of Ω^* is the closure of a simply connected domain and contains a rectifiable Jordan curve Γ such that the linear measure of*

$\Gamma \cap \mathbb{T}$ is positive, then the Taylor shift on $A^p(\Omega)$ is topologically mixing. If, in addition, $\mathbb{D} \subset \Omega$ then T is not frequently hypercyclic for any $p \geq 2$.

REMARK 3.2. Theorem 3.1 extends a corresponding result in [5], where Ω is a Caratheodory domain that contains a (nontrivial) subarc of $\mathbb{T}$. It shows, in particular, that in the situation of Example 2.13 the Taylor shift on $A^p(\Omega)$ is topologically mixing for all $p \geq 1$ and not frequently hypercyclic for any $p \geq 2$.

According to Example 2.10, for each (nontrivial) arc $B \subset \mathbb{T}$ the Taylor shift on $A^p(\mathbb{C}_\infty \setminus B)$ is topologically mixing for $p < 2$. We do not know if this is still the case for $p \geq 2$.

The remaining part of the section is devoted to the proof of Theorem 3.1. Our aim is to apply a version of Kitai's Criterion (see [13, Remark 3.13]).

Let $E \subset \mathbb{C}$ be compact and let $M(E)$ denote the set of complex measures on the Borel sets of $\mathbb{C}$ with support in E. It turns out that the Cauchy transforms of measures $\mu \in M(\Omega^*)$ are of particular interest for analysing the Taylor shift on $A^p(\Omega)$. For $\mu \in M(E)$ the Cauchy transform $C\mu \in H(E^*)$ of μ is defined (in terms of vector valued integration) by

$$C\mu := \int \gamma(\zeta)\, d\overline{\mu}(\zeta) = \int \frac{1}{1-\zeta\,\cdot}\, d\overline{\mu}(\zeta)$$

We write $|\mu|$ for the total variation of the measure μ and set

$$\mathcal{M}_p(\Omega) = \{\mu \in M(\Omega^*) : \int |\gamma(\zeta)|\, d|\mu|(\zeta) \in L^p(\Omega)\}$$

as well as

$$\mathcal{C}_p(\Omega) = \{C\mu \colon \mu \in \mathcal{M}_p(\Omega)\}.$$

For $f \in \mathcal{C}_p(\Omega)$ we denote by $\mathcal{C}^{-1}(f) = \{\mu \in \mathcal{M}_p(\Omega) : C\mu = f\}$ the set of representing measures for f. Note that for $\mathcal{C}_p(\Omega) \subset A^p(\Omega)$ since Cauchy transforms of measures $\mu \in \mathcal{M}_p(\Omega)$ are holomorphic in Ω and

$$|C\mu| \leq \int |\gamma(\zeta)|\, d|\mu|(\zeta) \in L^p(\Omega).$$

LEMMA 3.3. *Let $1 \leq p < \infty$ and $0 \in \Omega \subset \mathbb{C}_\infty$ be an open set. Then $T(\mathcal{C}_p(\Omega)) \subset \mathcal{C}_p(\Omega)$ and for $R \colon \mathcal{M}_p(\Omega) \to \mathcal{M}_p(\Omega)$, defined by $d(R\mu)(\zeta) = \overline{\zeta}\, d\mu(\zeta)$, the diagram*

$$\begin{array}{ccc} \mathcal{M}_p(\Omega) & \xrightarrow{R} & \mathcal{M}_p(\Omega) \\ {\scriptstyle C}\downarrow & & \downarrow{\scriptstyle C} \\ \mathcal{C}_p(\Omega) & \xrightarrow[T]{} & \mathcal{C}_p(\Omega) \end{array}$$

commutes.

Proof. Let $1 \le p < \infty$ and $\mu \in \mathcal{M}_p(\Omega)$. We first show that R is a self map. For $c := \max_{z \in \partial\Omega^*} |z|$ we obtain

$$\int |\gamma(\zeta)| d|R\mu|(\zeta) = \int |\gamma(\zeta)\zeta| d|\mu|(\zeta) \le c \int |\gamma(\zeta)| d|\mu|(\zeta).$$

It follows that $R\mu \in \mathcal{M}_p(\Omega)$. Now, let $f \in \mathcal{C}_p(\Omega)$ with $\mu \in \mathcal{C}^{-1}(f)$. Since we can interchange integration and T (cf. [21, Exercise 3.24]), we obtain

$$Tf = \int T\gamma(\zeta) d\overline{\mu}(\zeta) = \int \zeta\gamma(\zeta) d\overline{\mu}(\zeta) = \int \frac{\zeta}{1-\zeta\cdot} d\overline{\mu}(\zeta),$$

i.e. $Tf = CR\mu$. Since R is a self map on $\mathcal{M}_p(\Omega)$, it follows that $Tf \in \mathcal{C}_p(\partial\Omega^*)$. ∎

Inductively, from Lemma 3.3 we obtain

$$T^n f = \int \zeta^n \gamma(\zeta) d\overline{\mu}(\zeta) \tag{3.1}$$

for $f \in \mathcal{C}_p(\Omega)$, $\mu \in \mathcal{C}^{-1}(f)$ and $n \in \mathbb{N}$. In view of Kitai's criterion, our aim is to find measures μ such that $T^n(C\mu)$ converges to 0 in $A^p(\Omega)$. We shall see that this is the case if $\mu \in \mathcal{M}_p(\Omega)$ is supported on $\Omega^* \cap \mathbb{T}$ and a Rajchman measure. We recall that a Borel measure ν supported on $\mathbb{T}$ is called a Rajchman measure if the Fourier-Stieltjes coefficients $\hat{\nu}(k) = \int \zeta^k \, d\nu(\zeta)$ tend to 0 as k tends to $\pm\infty$ (see e.g. [16]).

Again according to Kitai's criterion, we also need a kind of right inverse of T: If $\mu \in \mathcal{M}_p(\Omega)$ is a measure with support in $\mathbb{T}$ we define

$$S_n\mu := \int \frac{\gamma(\zeta)}{\zeta^n} d\overline{\mu}(\zeta) = \int \frac{d\overline{\mu}(\zeta)}{\zeta^n(1-\zeta\cdot)} \tag{3.2}$$

for all $n \in \mathbb{N}$. As in the proof of Lemma 3.3 it is seen that $S_n\mu \in \mathcal{C}_p(\Omega)$.

LEMMA 3.4. *Let $1 \le p < \infty$ and $0 \in \Omega \subset \mathbb{C}_\infty$ be an open set which is either bounded in $\mathbb{C}$ or contains ∞. Furthermore, let T be the Taylor shift operator on $A^p(\Omega)$. If $f \in \mathcal{C}_p(\Omega)$ such that f is represented by a Rajchman measure $\mu_f \in \mathcal{C}^{-1}(f)$ supported on $\Omega^* \cap \mathbb{T}$ then $T^n f \to 0$ and $S_n\mu_f \to 0$ in $A^p(\Omega)$ as $n \to \infty$.*

Proof. Let $f \in \mathcal{C}_p(\Omega)$ and $\mu_f \in \mathcal{C}^{-1}(f)$ such that μ_f is a Rajchman measure supported on $B := \Omega^* \cap \mathbb{T}$. We fix $z \in \Omega$. By (3.1) we have

$$T^n f(z) = \int_{\partial\Omega^*} \frac{\zeta^n}{1-\zeta z} d\overline{\mu_f}(\zeta) = \int_B \frac{\zeta^n}{1-\zeta z} d\overline{\mu_f}(\zeta)$$

for all $n \in \mathbb{N}$. Because $\mu_f \in \mathcal{M}_p(\Omega)$ is supported on $B \subset \mathbb{T}$, the function $\gamma(z)$ belongs to $L^1(\mathbb{T}, |\mu_f|)$. Since μ_f is a Rajchman measure and $\mu_{f,z}$ with

$$d\mu_{f,z} := \gamma(z)\, d\overline{\mu_f}$$

is absolutely continuous with respect to $\overline{\mu_f}$, [16, Lemma 4, p. 77] yields that $\mu_{f,z}$ is a Rajchman measure as well. Thus, we have

$$T^n f(z) = \int \zeta^n \gamma(z)(\zeta) d\overline{\mu_f}(\zeta) = \int \zeta^n d\mu_{f,z}(\zeta) = \hat{\mu}_{f,z}(-n) \to 0$$

and

$$S_n \mu_f(z) = \int \zeta^{-n} \gamma(z)(\zeta) d\overline{\mu_f}(\zeta) = \int \zeta^{-n} d\mu_{f,z}(\zeta) = \hat{\mu}_{f,z}(n) \to 0$$

as n tends to ∞. Furthermore, for all $n \in \mathbb{N}$ we have

$$|T^n f(z)| \leq \int |\gamma(\zeta)| \, d|\mu_f|(\zeta) \quad \text{and} \quad |S_n \mu_f(z)| \leq \int |\gamma(\zeta)| \, d|\mu_f|(\zeta)$$

where $\int_B |\gamma(\zeta)| \, d|\mu_f|(\zeta)$ is p-integrable on Ω by assumption. Lebesgue's theorem of dominated convergence yields that $\|T^n f\|_p \to 0$ and $\|S_n \mu_f\|_p \to 0$ as n tends to ∞. ∎

As noted in the introduction, for $p \geq 2$ and $\zeta \in \partial\Omega^*$ the functions $\gamma(\zeta)$ are in general not p-integrable. We introduce appropriate means of the $\gamma(\zeta)$ which turn out to be integrable for all p.

REMARK 3.5. It is easily seen (see e.g. [15, Theorem 1.7]) that

$$\int_{\mathbb{T}} \frac{dm(\alpha)}{|1 - \alpha z|} = O\left(\log \frac{1}{1 - |z|}\right) \quad (|z| \to 1^-).$$

Since, for all $p \geq 1$,

$$\int_{\mathbb{D}} \left| \log \frac{1}{1 - |z|} \right|^p dm_2(z) \leq \frac{1}{\pi} \int_0^1 \left| \log \frac{1}{1 - r} \right|^p dr < \infty,$$

by symmetry we obtain that

$$\int |\gamma(\alpha)| dm(\alpha) \in L^p(\mathbb{C}_\infty \setminus \mathbb{T}, m_2).$$

For a Borel set $B \subset \mathbb{T}$ we define $dm_B = 1_B dm$ and

$$f_B := C m_B = \int \gamma(\alpha) \, dm_B(\alpha) = \int_B \frac{dm(\alpha)}{1 - \alpha \,\cdot} \in H(\overline{B}^*). \tag{3.3}$$

Let now $\Omega \subset \mathbb{C}_\infty$ be an open set and $\Omega^* \cap \mathbb{T} \neq \emptyset$. Then, for all Borel sets $B \subset \Omega^* \cap \mathbb{T}$ and for all $1 \leq p < \infty$

$$\int |\gamma(\alpha)| \, dm_B(\alpha) \in L^p(\Omega, m_2),$$

which yields that m_B is a measure in $\mathcal{M}_p(\Omega)$ supported on B and hence $f_B \in \mathcal{C}_p(\Omega)$. Since $1_B \in L^1(\mathbb{T})$ and the arc length measure is a Rajchman measure, Theorem [16, Lemma 4, p. 77] yields that m_B is a Rajchman measure as well.

The following result shows that under the conditions of Theorem 3.1 the functions f_B densely span $A^p(\Omega)$.

THEOREM 3.6. *Let $1 \leq p < \infty$ and $0 \in \Omega \subset \mathbb{C}_\infty$ be a domain which is either bounded in $\mathbb{C}$ or contains ∞. If each component K of Ω^* is the closure of a simply connected domain and contains a rectifiable Jordan curve Γ such that the linear measure of $\Gamma \cap \mathbb{T}$ is positive, then the span of $\{f_B : B \subset \Omega^* \cap \mathbb{T}\}$ is dense in $A^p(\Omega)$.*

Proof. We first assume that Ω is bounded in $\mathbb{C}$ and fix $\ell \in A^p(\Omega)'$ with $\ell(f_B) = 0$ for all Borel sets $B \subset \Omega^* \cap \mathbb{T}$. Again, according to the Hahn-Banach theorem there exists a function $g \in L^q(\Omega)$, where q is the conjugated exponent, such that

$$\ell(f) = \int_\Omega f\overline{g}\, dm_2$$

for all $f \in A^p(\Omega)$, and for $1_\Omega g dm_2 \in M(\overline{\Omega})$ the Cauchy transform

$$(Vg)(\alpha) = \int_\Omega \frac{\overline{g}(\zeta)}{1-\zeta\alpha} dm_2(\zeta)$$

of $1_\Omega g dm_2$ is holomorphic in the interior of Ω^*. However, since $q \leq 2$ for $p \geq 2$, it is no longer guaranteed that the Cauchy integral is defined and continuous on $\mathbb{C}$.

Since $\int_{\Omega^* \cap \mathbb{T}} |\gamma(\alpha)|\, dm(\alpha) \in L^p(\Omega)$, Hölder's inequality yields

$$\int_\Omega \int_{\Omega^* \cap \mathbb{T}} |\frac{g(\zeta)}{1-\zeta\alpha}|\, dm(\alpha)\, dm_2(\zeta) \leq \|g\|_q \cdot \| \int_{\Omega^* \cap \mathbb{T}} |\gamma(\alpha)|\, dm(\alpha)\|_p < \infty.$$

Hence the maximal Cauchy transform

$$\int_\Omega \frac{|g(\zeta)|}{|1-\zeta\alpha|} dm_2(\zeta)$$

is finite for m-almost all on $\alpha \in \Omega^* \cap \mathbb{T}$ and Vg exists m-almost everywhere on $\Omega^* \cap \mathbb{T}$. Moreover, for all Borel sets $B \subset \Omega^* \cap \mathbb{T}$ we may apply Fubini's theorem to get

$$0 = \ell(f_B) = \int_\Omega \int_B \frac{dm(\alpha)}{1-\zeta\alpha}\, \overline{g}(\zeta)\, dm_2(\zeta) = \int_B Vg(\alpha)\, dm(\alpha).$$

This implies that $Vg = 0$ m-almost everywhere on $\Omega^* \cap \mathbb{T}$.

Let G be a bounded simply connected domain in $\mathbb{C}$ and let $D_q(G)$ denote the the Dirchlet space of order q defined as in Remark 2.12. Fixing a point $\beta \in G$, we equip $D_q(G)$ with the (complete) norm

$$\|h\|_q = |h(\beta)| + \Big(\int_G |h'|^q\, d\lambda_2 \Big)^{1/q}.$$

If φ is the conformal mapping from $\mathbb{D}$ to G with $\varphi(0) = \beta$ and $\varphi'(0) > 0$ then

$$h \mapsto (h \circ \varphi)(\varphi')^{2-q}$$

defines an isomorphism between $D_q(G)$ and the Dirichlet space $D_q := D_q(\mathbb{D})$ on the unit disc. It is known that $D_q \subset H^q$, where H^q denotes the Hardy space of order q (see e.g. [7, p. 88]). In particular, for $h \in D_q(G) \subset D_1(G)$ we have

$(h \circ \varphi)\varphi' \in H^1$, which in turn implies that h belongs to the Hardy-Smirnov space $E^1(G)$ (see [8, Corollary to Theorem 10.1]).

Let now K be a component of Ω^* and G the interior of Γ. Then the harmonic measure $\omega(\cdot, K \cap \mathbb{T}, G)$ is positive or G meets $\mathbb{T}$. In a similar way as in the proof of Theorem 1 in [19], by applying Theorem 3, Chapter II, Section 4, from [22] it can be shown that $Vg|_G$ belongs to $D_q(G)$. Since $Vg = 0$ m-almost everywhere on $\Omega^* \cap \mathbb{T}$, in the case of positive harmonic measure $\omega(\cdot, K \cap \mathbb{T}, G)$ the local F. and M. Riesz theorem implies that Vg vanishes on a subset of $K \cap \mathbb{T}$ of positive harmonic measure (cf. Remark 2.12).

Since $\Gamma := \partial G$ is a rectifiable Jordan curve, the set of cone points of Γ has full linear measure (see [12, Corollary 1.3 and p. 207] or [8, Section 3.5]). Hence m-almost every point in $K \cap \mathbb{T}$ is a cone point. it With similar arguments as in the proof of Theorem 3.2.4 in [10] it can be shown that the non-tangential limit of Vg at α coincides with $(Vg)(\alpha)$ and hence with 0 at m-almost all $\alpha \in K \cap \mathbb{T}$. From [8, Theorem 10.3] we obtain that $Vg|_G = 0$. As K was an arbitrary component of Ω^*, it follows that $Vg|_{(\Omega^*)^\circ} = 0$ and then

$$\ell(\gamma(\alpha)) = (Vg)(\alpha) = 0$$

for all $\alpha \in (\Omega^*)^\circ$. Since, by assumption, Ω is a domain, it follows that the inner boundary of $\overline{\Omega}$ is empty, which allows to apply [14, Corollary p. 162] showing that the rational functions with (simple) poles in $\mathbb{C} \setminus \overline{\Omega}$ are dense in $A^p(\Omega)$. This implies that $\ell = 0$ and thus the denseness of span$\{f_B : B \subset \Omega^* \cap \mathbb{T}\}$ in $A^p(\Omega)$.

Along the same lines, we get in case of Ω containing ∞ and $\Omega_\rho := \Omega \cap \rho\mathbb{D}$ that the span of $\{f_B : B \subset \Omega^* \cap \mathbb{T}\} \cup \gamma(\rho^{-1}\mathbb{D})$ is dense in $A^p(\Omega_\rho)$. According to Remark 2.6, the span of $\{f_B : B \subset \Omega^* \cap \mathbb{T}\}$ is dense in $A^p(\Omega)$. ∎

Proof of Theorem 3.1. By Theorem 3.6, the span L of $\{f_B : B \subset \Omega^* \cap \mathbb{T}\}$ is dense in $A^p(\Omega)$. Furthermore, for $f = \sum_B \lambda_B f_B \in L$ let $(S_n\mu_f)_{n\in\mathbb{N}}$ be the sequence defined in (3.2) with $\mu_f = \sum_B \lambda_B m_B$. Then Lemma 3.4 yields that $\|T^n f\|_p$ and $\|S_n\mu_f\|_p$ converge to 0 as $n \to \infty$. Hence, applying Lemma 3.3 and interchanging integration and T^n we obtain for $n \in \mathbb{N}$

$$T^n S_n \mu_f = T^n \left(\int \gamma(\zeta)\zeta^{-n}\, dm(\zeta) \right) = \int T^n \gamma(\zeta)\zeta^{-n}\, dm(\zeta) = f.$$

The Kitai criterion in the version [13, Exercise 3.1.1 or Remark 3.13] yields the assertion.

Finally, the denseness of the span of $\{f_B : B \subset \Omega^* \cap \mathbb{T}\}$ implies that in the case $\mathbb{D} \subset \Omega$ and $p \geq 2$ the space $A^p(\Omega)$ is (continuously and) densely embedded in $A^2(\mathbb{D})$. Since the Taylor shift is not frequently hypercyclic on $A^2(\mathbb{D})$ ([13, Example 9.18]) it is also not frequently hypercyclic on $A^p(\Omega)$. ∎

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JÜRGEN MÜLLER, FB IV, MATHEMATICS, UNIVERSITY OF TRIER, D-54286 TRIER, GERMANY
E-mail address: jmueller@uni-trier.de

MAIKE THELEN, FB IV, MATHEMATICS, UNIVERSITY OF TRIER, D-54286 TRIER, GERMANY
E-mail address: maikethelen@web.de